SYMMETRY ENERGY OF THE SPIN-POLARIZED NUCLEAR MATTER

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Abstract: In this report, we present the results of a mean-field study for the equation of state (EOS) of spin polarized nuclear matter. The realistic choices of the effective, density-dependent nucleon-nucleon (NN) interactions that well behave in different nuclear structure and reaction studies have been used within the non-relativistic Hartree-Fock formalism. We examined the density dependence of the symmetry energy and its role onto the mass and radius of the neutron star.

Tóm tắt: Báo cáo trình bày kết quả các tính toán trường trung bình cho phương trình trạng thái của chất hạt nhân phân cực spin. Trong gần đúng Hartree-Fock phi tương đối tính, chúng tôi sử dụng các tương tác hiệu dụng phụ thuộc mật độ đã được sử dụng thành công trong nhiều công trình mô tả cấu trúc và phản ứng hạt nhân. Báo cáo xem xét sự phụ thuộc mật độ của năng lượng đối xứng và vai trò của đại lượng này lên khối lượng và bán kính của sao neutron.

Keywords: *spin-polarized nuclear matter, symmetry energy, neutron star, effective NN interaction*

1. INTRODUCTION

The effects of magnetic fields on dense matter have been a subject of interest for a long time [1] particularly in relation to astrophysical issues. Pulsars are believed to be rapidly rotating neutron stars endowed with strong magnetic fields [2]. Numerous magnetars (neutron stars with an extremely powerful magnetic field that emit high-energy electromagnetic radiation in the form of X-rays and gamma rays) have been detected [3], and the astrophysics studies of NS or PNS being in a strong magnetic field are now of high interest.

Under the strong magnetic filed, the matter becomes spin-polarized. The equation of state (EOS) for spin-polarized matter is important for the neutron star structure and for the cooling of magnetized stars [1]. The knowledge of the density dependence of $E_{\mathrm{sym}}(\rho)$ is extremely important for the construction of nuclear EOS. $E_{\mathrm{sym}}(\rho)$ can affect the EOS of the β-stable neutron star (NS) matter as well as the main NS properties like the maximum mass, radius, central density, and moment of inertia. It has been, therefore, a longstanding goal of many nuclear structure and reaction studies. The properties of spin-polarized nuclear matter have been studied by Vidaña *et al.* within the Brueckner-Hartree-Fock (BHF) approximation [4] however the behavior of the symmetry energy and its effect to NS did not clarified.

In this work we study the symmetry energy properties of spin-polarized isospin asymmetric nuclear matter up to the high density region relevant for neutron stars. We consider a pure nucleonic description of dense matter, although additional degrees of freedom, such as hyperons, kaon or pion condensates, or quark matter could be present at such high densities. Our calculations are based on the non relativistic Hartree-Fock (HF) mean field using the realistic interaction between nucleons in the high-density nuclear medium. The medium effects are normally considered as the physics origin of the density dependence that were added to the M3Y interactions [5] to properly account for the NM saturation and reproduce numerous nucleon-nucleus and nucleus-nucleus scattering data [6,7]. This versions of the density-dependent M3Y interaction have been used recently to study the basic properties of asymmetric NM at zero and finite temperature as well as the β-stable matter of neutron star and proto-neutron star [8, 9]. The BHF results obtained by Isaac Vidaña and Ignazio Bombaci [4] for the energy density of the non-polarized symmetric nuclear matter have been parametrized in the same form of the density dependence M3Y interaction to observe the symmetry energy.

2. THE MAIN PART OF REPORT

2. 1. Hartree-Fock calculation of spin-polarized assymmetric nuclear matter

The equation of state (EOS) of nuclear matter at finite temperature is produced within Hartree-Fock (HF) mean field approximation. These are characterized by given values of proton and neutron up and down densities, or equivalent by total density $\rho = \rho_n + \rho_p$, neutron-proton asymmetry $\delta = (\rho_n - \rho_p)=\rho$ and spin polarization $\Delta_n = (\rho_{\uparrow n} - \rho_{\downarrow n})\rho_n$; $\Delta_p = (\rho_{\downarrow p} - \rho_{\downarrow p})\rho_p$. The following expression are being used in the formular of HF calculation.

$\rho_n + \rho_p = \rho_\uparrow + \rho_\downarrow = \rho; \rho_n - \rho_p = \delta; \rho_{\uparrow n} + \rho_{\uparrow n} = \rho_n ; \rho_{\uparrow n} - \rho_{\downarrow n} = \Delta_n\rho_n; \rho_{\uparrow p} + \rho_{\downarrow p} = \rho_p; \rho_{\uparrow p} - \rho_{\downarrow p} = \Delta_p\rho_p.$

The Fermi momentum $k^{\sigma\tau}_F$ is related to the corresponding partial density by $k^{\sigma\tau}_F = (6\pi^2\rho_{\sigma\tau})^{1/3}$ with $\tau=n,p$ and $\sigma=\uparrow,\downarrow$.

The total energy density is determined as:

$$E(\rho, \delta, \Delta_n, \Delta_p)=E_{kin}(\rho, \delta, \Delta_n, \Delta_p)+E_{pot}(\rho, \delta, \Delta_n, \Delta_p) \tag{1}$$

$$E_{\text{kin}}(\rho, \delta, \Delta_n, \Delta_p) = \sum_{k\sigma\tau} \frac{\hbar^2 k^2}{2m_\tau}, \tag{2}$$

where m_τ is the nucleon mass.

$$E_{\text{pot}}(\rho, \delta, \Delta_n, \Delta_p) = \frac{1}{2} \sum_{k\sigma\tau} \sum_{k'\sigma'\tau'} [\langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau' | v_{\text{D}} | \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau' \rangle + \langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau' | v_{\text{EX}} | \boldsymbol{k}'\sigma\tau, \boldsymbol{k}\sigma'\tau' \rangle] \tag{3}$$

The single-particle wave function $|\boldsymbol{k}\sigma\tau\rangle$ is a plane wave: $|\boldsymbol{k}\sigma\tau\rangle \sim e^{i\boldsymbol{kr}}\chi_\sigma\chi_\tau$ where χ_σ; χ_τ is Dirac spinor associated with spin and iso-spin components in a nucleon wave function. υ_D and υ_{EX} are the *Direct* and *Exchange* parts of the effective nucleon-nucleon interaction:

$$v_{\text{D(EX)}}(r) = v_{00}^{\text{D(EX)}}(r) + v_{10}^{\text{D(EX)}}(r)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + v_{01}^{\text{D(EX)}}(r)\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + v_{11}^{\text{D(EX)}}(r)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2). \tag{4}$$

The kinetic and potential energy density per particle are calculated as:

$$\frac{E_{kin}}{A} = \frac{3\hbar^2}{10m} \frac{(k_F^{\uparrow n})^2 \rho_{\uparrow n} + (k_F^{\downarrow n})^2 \rho_{\downarrow n} + (k_F^{\uparrow p})^2 \rho_{\uparrow p} + (k_F^{\downarrow p})^2 \rho_{\downarrow p}}{\rho} \tag{5}$$

$$\varepsilon_{\rm pot}(\rho,\delta,\Delta_n,\Delta_p) = \varepsilon_{00}+\varepsilon_{10}+\varepsilon_{01}+\varepsilon_{11} \tag{6}$$

$$\varepsilon_{00} = \frac{1}{2}J_{00}^D F_{00}(\rho)\rho + \frac{1}{2\rho}F_{00}(\rho)\int A_{00}^2(r)v_{00}^{\rm EX}(r)d^3r$$
$$\varepsilon_{10} = \frac{1}{2}J_{10}^D F_{10}(\rho)\left(\Delta_n\frac{1+\delta}{2}+\Delta_p\frac{1-\delta}{2}\right)^2\rho + \frac{1}{2\rho}F_{10}(\rho)\int A_{10}^2(r)v_{10}^{\rm EX}(r)d^3r$$
$$\varepsilon_{01} = \frac{1}{2}J_{10}^D F_{01}(\rho)\delta^2\rho + \frac{1}{2\rho}F_{01}(\rho)\int A_{01}^2(r)v_{01}^{\rm EX}(r)d^3r$$
$$\varepsilon_{11} = \frac{1}{2}J_{11}^D F_{11}(\rho)\left(\Delta_n\frac{1+\delta}{2}-\Delta_p\frac{1-\delta}{2}\right)^2\rho + \frac{1}{2\rho}F_{11}(\rho)\int A_{11}^2(r)v_{11}^{\rm EX}(r)d^3r$$

where $J_{ST} = \int d^3r\, v_{ST}$ and

$$A_{00}(r) = \rho_{\uparrow n}\hat{j}_1(k_{F_{\uparrow n}}r)+\rho_{\downarrow n}\hat{j}_1(k_{F_{\downarrow n}}r)+\rho_{\uparrow p}\hat{j}_1(k_{F_{\uparrow p}}r)+\rho_{\downarrow p}\hat{j}_1(k_{F_{\downarrow p}}r)$$
$$A_{10}(r) = \rho_{\uparrow n}\hat{j}_1(k_{F_{\uparrow n}}r)-\rho_{\downarrow n}\hat{j}_1(k_{F_{\downarrow n}}r)+\rho_{\uparrow p}\hat{j}_1(k_{F_{\uparrow p}}r)-\rho_{\downarrow p}\hat{j}_1(k_{F_{\downarrow p}}r)$$
$$A_{01}(r) = \rho_{\uparrow n}\hat{j}_1(k_{F_{\uparrow n}}r)+\rho_{\downarrow n}\hat{j}_1(k_{F_{\downarrow n}}r)-\rho_{\uparrow p}\hat{j}_1(k_{F_{\uparrow p}}r)-\rho_{\downarrow p}\hat{j}_1(k_{F_{\downarrow p}}r)$$
$$A_{11}(r) = \rho_{\uparrow n}\hat{j}_1(k_{F_{\uparrow n}}r)-\rho_{\downarrow n}\hat{j}_1(k_{F_{\downarrow n}}r)-\rho_{\uparrow p}\hat{j}_1(k_{F_{\uparrow p}}r)+\rho_{\downarrow p}\hat{j}_1(k_{F_{\downarrow p}}r)$$

with $\hat{j}_1(x) = 3j_1(x)/x$ where $j_i(x)$ is the ith-order spherical Bessel function.

The density dependence of the total energy of NM (per baryon) is usually expressed as

$$E/A\,(\rho,\delta) = E/A\,(\rho;\delta=0) + E_{sym}(\rho)\delta^2 + O(\delta^4) + \dots \tag{7}$$

$E_{sym}(\rho)$ is the so-called **symmetry** energy.

- **Effective density-dependent NN interaction**

Because the HF method is the first order of many-body calculations, it is necessary to have an appropriate in-medium NN interaction. The evaluating an in-medium NN interaction started from the free NN interaction still remains a challenge for the nuclear many-body theory. In the present research, we use 2 different density-dependent versions of the M3Y interaction. One have been used in the HF calculation of symmetric and asymmetric NM [5] and in numerous folding model studies of the nucleon-nucleus and nucleus-nucleus scattering [6, 7] - called CDM3Y6. One has been parametrized to reproduce simultaneously the BHF results obtained by Isaac Vidaña and Ignazio Bombaci [4] for the energy density of the non-polarized symmetric nuclear matter (Δ_n=0, δ=0) (the solid line in Fig. 1a), non-polarized neutron matter (Δ_n=0, δ=1) (the solid line in Fig. 1c), and total-polarized symmetric nuclear matter (Δ_n=Δ_p=1, Δ_n=-Δ_p=1, δ=0) (the dashed and dotted lines in Fig 1a). We use the same form of coupling constants $F_{00}(\rho)$, $F_{10}(\rho)$, $F_{01}(\rho)$, $F_{11}(\rho)$ dependent on densities in these two M3Y effective interactions:

$$v_{\rm D,(EX)}(\rho,r) = F_{00}(\rho)v_{00}^{\rm D(EX)}(r)+F_{10}(\rho)v_{10}^{\rm D(EX)}(r)+F_{01}(\rho)v_{01}^{\rm D(EX)}(r)+F_{11}(\rho)v_{11}^{\rm D(EX)}(r) \tag{8}$$

$$F_{00}(\rho)=C_{00}[1+\alpha_{00}\exp(-\beta_{00}\rho)+\gamma_{00}\rho] \quad (9)$$

The parameter sets *Cj; αj; βj; γj* for all the *spin* and *isospin* components are presented in Table 2.1. The sets for CDM3Y-BHF taken in 4 cases of spin-polarized nuclear matter also give the good fit to BHF energy density in other cases as shown in Fig 1(b,c).

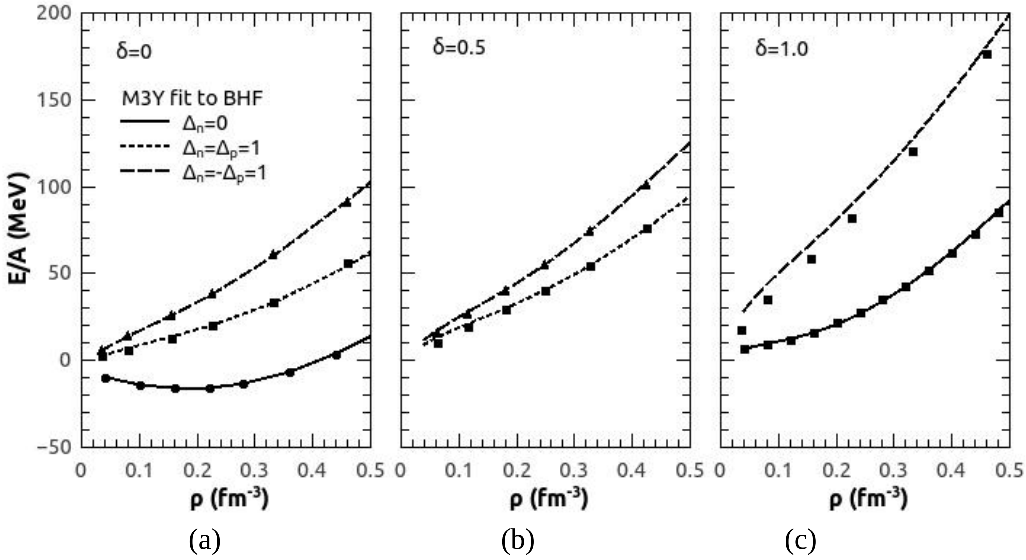
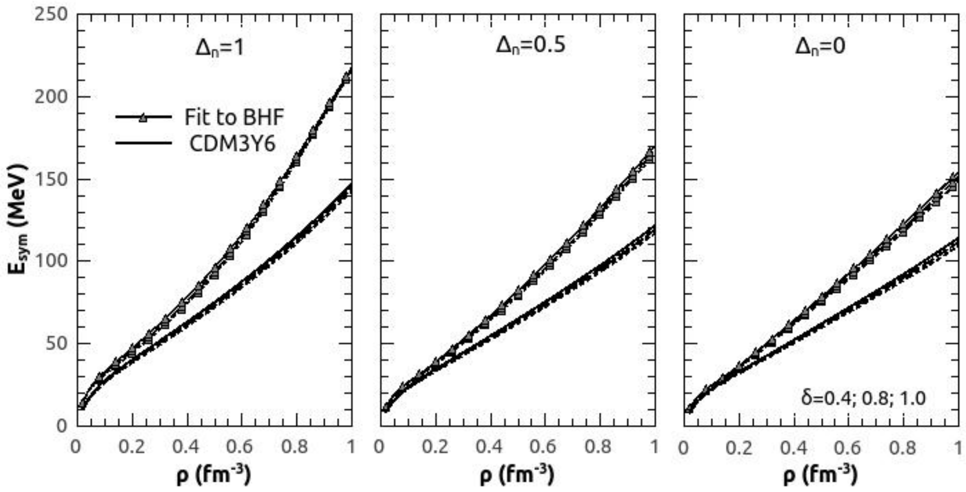


Figure 1. *Total nuclear matter energy per particle* E/A *at different neutron-proton asymmetries* δ and polarization parameters Δ_n, Δ_p *given by the Hartree-Fock calculations using the CDM3Y-BHF (which is fitted to BHF in (a) and solid line in (c)). The scatter points are results of the BHF calculation by* Vidaña and Bombaci *from [4].*

Table 2.1: Parameters of the density dependence of CDM3Y6 and CDM3Y-BHF interaction

Interaction	ST	C	α	β (fm^3)	γ (fm^3)
CDM3Y6	00	0.2658	3.8033	1.4099	-4.0
	10	1.0	0.	0.	0.
	01	0.2313	7.6800	9.6498	6.7202
	11	1.0	0.	0.	0.
CDM3Y-BHF	00	1.1556	0.8275	26.2142	1.4342
	10	0.5148	5.1529	15.44	-3.9241
	01	3.1791	-0.0037	-5.2419	-1.6843
	11	1.2535	0.8377	13.5560	0.4051

2. 2. Results and discussion



(c) (b) (a)

Figure 2. HF results for the symmetry energies $E_{sym}(\rho)$ of the spin-polarized nuclear matter given by the density dependent NN interactions under study.

The HF results for the symmetry energy $E_{sym}(\rho)$ of the non-polarized ($\Delta_n = -\Delta_p = 0$), partly polarized ($\Delta_n = -\Delta_p = 0.5$), and totally polarized ($\Delta_n = -\Delta_p = 1$) nuclear matter given by the considered interactions have been plotted in Fig. 2. Since magnetic moment of neutron and proton are opposite in signs then we do **not** consider the $\Delta_n = \Delta_p$ cases. The quadratic law are usually probed in the earlier study [8]. In this report, we firstly examine this law in the cases of spin-polarized nuclear matter. It is obvious from Fig. 2 that the symmetry energy $E_{sym} = [E/A(\rho, \delta)-E/A(\rho, \delta=0)]/\delta^2$ of the NM at a given polarization does not depend on the isospin asymmetry parameter δ, imposes a quadratic dependence upon δ of the spin-polarized nuclear matter energy: $E/A(\rho, \delta) = E/A(\rho, \delta=0) + E_{sym}(\rho)\ \delta^2$. Secondly, we tracking the changes of nuclear symmetry energy by polarization parameter Δ_n, Δ_p. In this report we only consider $\Delta_n = -\Delta_p$ case. One can see from Fig 2c to 2a, as the polarization increases, the energy per particle of the polarized symmetrical nuclear matter increases or *stiffer*. This can result in the equation of state of the beta-stable matter in the outer-core of a neutron star then affect the properties of the star, such as the mass and radius shown in Fig. 3. We often know that the stiffer EOS describes the more compact neutron star matter, therefore it should gives the heavier mass M and/or smaller radius R. However as seen in Fig. 3, the stiffer EOS (given by higher polarization) gives the smaller mass and larger radius, one can say it not easy to observe the effect of symmetry energy through the mass-radius relation of the star.

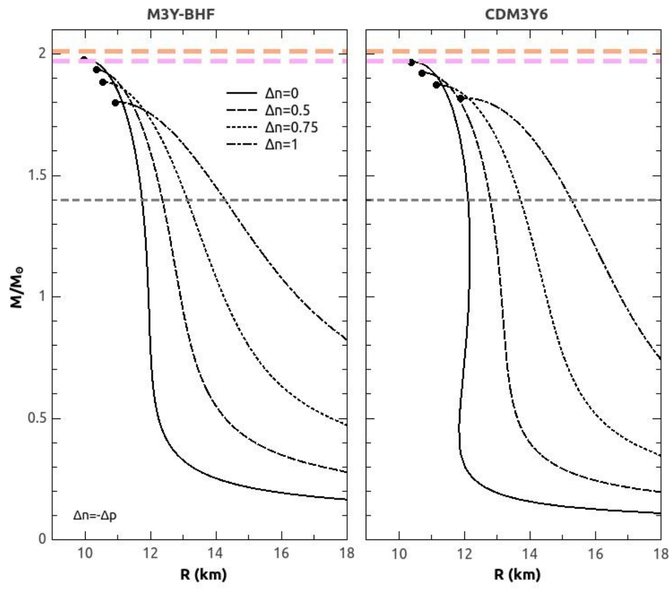


Figure 3. Mass versus radius obtained with the EOS's given by the considered NN interaction, in comparison with the empirical data (dashed thick line) from recent astronomical observations of neutron stars [10, 11]

3. CONCLUSION

Using the Hartree-Fock methodology with the effective nucleon-nucleon interaction of CDM3Y6 and the one fitted to BHF results, we have computed the symmetry energy properties of polarized asymmetric nuclear matter, that is related with the configuration of a static neutron star. We have also seen that total energy per particle is parabolic on the isospin asymmetric δ in a very good approximation up to full polarization for all densities. In conclusion, we see that the equation of state of polarized symmetrical nuclear matter becomes stiffer by increasing the polarization in the density range which was considered, however, the effect of symmetry energy to the properties of neutron star must be examined further than mass and radius.

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