COMPARATIVE ANALYSIS OF THE DIFFERENTIAL EVOLUTION AND GENETIC ALGORITHM APPLIED TO THE NUCLEAR REACTOR FUEL RELOADING OPTIMIZATION

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Abstract: In-core fuel management optimization (ICFMO) is a prominent problem in the nuclear engineering field. This is a multi-objective problem with large combinatorial solution space, multiple conflicting, nonlinear objective functions. In this paper, two meta-heuristic approaches, Differential evolution and Genetic algorithms were proposed to solve this problem. The optimal objectives are both the maximization of the length and the minimization of power peaking factor in a fuel cycle. The algorithms were applied to the Dalat nuclear research reactor (DNRR). Comparative analysis between two methods and with the actual work configuration was conducted. The results demonstrate that the performance of both algorithms is satisfactory and the DE is more effective than the GA in the DNRR case.

Keywords: Differential evolution algorithm, genetic algorithm, fuel reloading optimization, nuclear reactor, DNRR

1. INTRODUCTION

In the ICFMO problem, reloading and reshuffling of fuel assemblies in the nuclear reactor core is an optimization problem which means to find the best configuration of shuffling between fresh fuel and remnants from the previous cycle. It is known as a multi-objective problem including the maximization of end of cycle reactivity, discharged burn-up and the minimization of power peaking, feed enrichment, the burnable poison inventory, where an improvement in one object may deteriorate another [1]. The obtained result is not the single best solution, but a set of solutions that are equally good. Traditional optimization techniques, such as out-in pattern, scatter loading, out-in scatter procedure [2] and the bi-directional axial [3] cannot result in adequate solutions. Heuristic search programs applied to solve this problem showed more efficiently than the traditional methods but they usually get trapped in a local optimum and thus fail to obtain the global optimum solution [4].

In recent years, the advanced meta-heuristic search approaches are being applied to many optimization problems due to their component of diversification that is used to explore the global search space to avoid being trapped in a local optimum. Evolutionary algorithms are classified under a family of algorithms for global optimization by the evolution of populations of individuals in nature [5]. They basically follow a specific strategy with different variations to select candidate elements from population set and apply crossover and mutations to modify the elements to improve the quality of modified elements. Among the many evolutionary algorithms, the two similar and popular are the genetic algorithm (GA) [6] and differential evolution (DE) [7] which are performed by three evolutionary operators: selection, crossover, and mutation. Most of the applications of GA are applied to the discrete variable optimization problems which contain the fuel loading optimization problem. Several studies of the application of GA to loading pattern design indicated the potential results to various reactors [8-11]. Otherwise, the DE algorithm which is originally developed for continuous variable problems was found several applications in fuel assembly and core design optimal problem [12-13]. Although DE has been successful in numerical optimization, few works concern its usage for discrete optimization problems as reloading and reshuffling of fuel assemblies in the nuclear reactor core.

In the present work, binary mixed integer coded GA and discrete DE algorithm have been developed and applied to the problem of fuel loading optimization of the Dalat nuclear research reactor (DNRR). The 3D finite difference multi-group diffusion theory code CITATION [14] was used in the analysis of neutronics characteristics providing the values of k_{eff} and power peaking factor to the changes in the fuel loading pattern (LP). The group constants are generated by using WIMSD-5B [15] with ENDF/B-VII.0 nuclear data library.

2. PROBLEM AND METHODOLOGY

2. 1 Multi-objective optimization problem of the DNRR

The fuel reloading optimization problem suggested for comparative analysis of the GA and new DE algorithms were performed to the Dalat nuclear research reactor (DNRR). This reactor operated with the nominal power of 500 kW, is an upgraded modification of 250 kW TRIGA Mark II since the 1980s. The reactor core loaded with the Russian fuel type VVR-M2, consists of 121 hexagonal cells of fuel bundles (FBs), control rods, irradiation channel, beryllium blocks, and aluminum chocks. The base LP was 100 FBs with 0 – 12.3% burn-up depicted in Figure 1. The maximum effective multiplication factor (k_{eff}) and minimum power peaking factor (PPF) were selected as the optimization criterions. According to the weighted sum method, the objective of the optimization problem was the maximum fitness function:

$$Fitness = \alpha \times (k_{eff} - 1) + \beta \times (PPF_0 - PPF) \quad (1)$$

where PPF_0 is an input factor that is chosen so that PPF is always lower than it; the coefficients α and β are the weighting factors for k_{eff} and PPF, respectively.

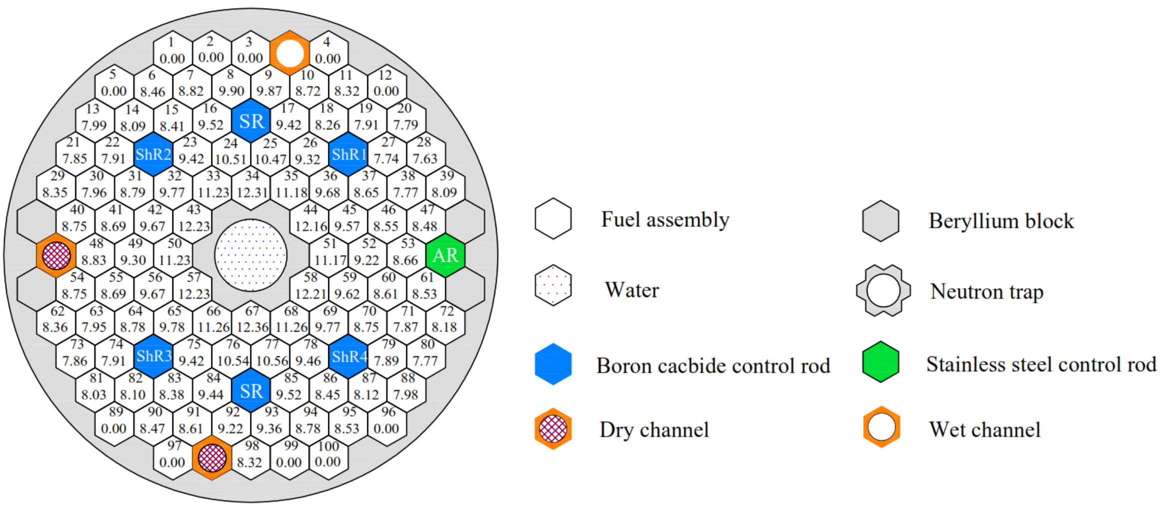


Fig. 1 Base loading pattern of the fuel reloading optimization problem of the DNRR.

In this work, a new method based on GA was developed in conjunction with the junction with the weighted sum method to automatically determine the weighting factors (α, β) in the objective function for the duration of the search process. The two search schemes of the weighting factors and the optimal fuel reloading patterns were implemented simultaneously. It is expected that the search direction is flexible and the search may move towards a set of approximate global optimal solutions. In the application of the proposed DE, PPF_0 = 2 is an input factor that is chosen, the coefficients α = 1000 and β = 100 are the weighting factors for k_{eff} and PPF, respectively. These values of α and β are already used in the research works on the reactor fuel reloading optimization [16-17] and are considered to be good enough to make the search process give good results.

2. 2 A binary mixed integer coded genetic algorithm

The proposed GA works with two types of chromosomes: integer chromosome and binary chromosome. The integer chromosome represents fuel LPs and the binary chromosome represents weighting factors. The two different kinds of genetic operators required to work on the two types of chromosomes are shown in the sections 2.2.2 and 2.2.4. The coding procedures are described in the sections 2.2.1 and 2.2.3.

2.2.1 Coding procedure for LPs

Consider a reactor core consisting of 100 positions for fuel loading and the total number of FBs loaded in the core is also equal to 100. First, number all the core positions by integers from 1to 100. An FB in the base LP is then assigned with the same number as the core position into which the FB is loaded as seen in Figure 1.

Encoding: an LP is encoded into a chromosome of length 100 that is a string of 100 integer numbers (i_1 i_2 ... i_{100}), where i_k= {1, ..., 100} is the FB number. The position of gene i_k in the chromosome defines the core position *pos(k)* into which the FB i_k is loaded.

Decoding: the chromosome (i_1 i_2 ... i_{100}) is decoded into an LP by loading FB number i_k into position number *pos(k)* in the reactor core.

2.2.2 Genetic operators for integer chromosomes

GA basically works with three genetic operators: selection, crossover, and mutation [6]. Selection carries better solutions into the next generation based on their fitness values. Crossover mixes parts of two parent solutions to create two different off-springs. Mutation makes some small random changes in the solutions maintaining the diversity of population to prevent a premature convergence to local optima.

In this study, the elitism strategy (ES) [18] is used in selection to preserve the best solutions during the search process. A solution *A* with $PPF_1$and k_{eff1} is dominated by a solution *B* with PPF_2 and k_{eff2} if $PPF_2 \leq PPF_1$ and $k_{eff\,2} \geq k_{eff\,1}$. Any solution that is not dominated by others is regarded as a non-dominated solution. The entire population can be ranked by sorting through to identify all non-dominated solutions in the archive. Every solution in the archive is directly transferred to a breeding pool for the next generation.

One of the most popular crossover method used in this work is the one-point method. For one dimensional integer chromosomes, the one-point crossover is performed by two steps: two members of the breeding pool be mated are randomly selected with a crossover probability, then the two chromosomes undergo crossing over as follows: an integer position k along the chromosome is selected uniformly at random between 1 and the chromosome length less one N –1. Two new strings are created by swapping all characters between positions k +1 and N inclusively. In case two parents have some genes with the same number, crossing these two parents over may create two off-springs which have some identical genes.

If this case occurs, small random numbers between 0 and 1 are added to the genes with the same value to make a difference between them, and then rank these genes again to make two new off-springs.

Mutation is conducted by a binary shuffle of two genes in the chromosome. A chromosome to be mutated is randomly selected with a mutation probability. Then two uniformly selected genes of the chromosome are exchanged their positions.

2.2.3 Coding procedure for weighting factors

Since the optimization problem in this study has two weighting factors and their sum is 1, only one of the weighting factors needs to be found in the search process. The other factor β is certainly defined by (1 – α). Below is the coding procedure for the factor α .

The factor α is encoded into a binary chromosome that is a string of bits 1 or 0. A chromosome is represented symbolically by the string of m_i: $(m_1 m_2 ... m_l)$ where m_i may take on a value 1or 0. The length of the string is determined by the precision of and it's limiting range. The length l is the minimum integer that satisfies the following formula:

$$\left(\alpha_{\max} - \alpha_{\min}\right) \times 10^n \le 2^l - 1 \qquad (2)$$

where $\alpha_{\min}$ and $\alpha_{\max}$ are the minimum and maximum values of α ,and n is the number of digits following the decimal point in the number that represents the value of α . As $0 < \alpha < 1$, the above equation becomes:

$$10^n \le 2^l - 1 \qquad (3)$$

The binary string is decoded into the real value of α based on the following formula:

$$\alpha = \alpha_{\min} + \frac{\alpha_{\max} - \alpha_{\min}}{2^l - 1} \sum_{i=1}^{l} m_i \times 2^{l-i} = \frac{1}{2^l - 1} \sum_{i=1}^{l} m_i \times 2^{l-i} \qquad (4)$$

2.2.4 Genetic operators for binary chromosomes

Genetic operators for the binary chromosomes in this study are similar to the genetic operators for integer chromosomes above. Selection is also performed by the roulette wheel spin method to create a breeding pool for the next generation. Crossover is performed by the one-point method to create two off-springs: First, a pair of strings in the breeding pool is randomly chosen with a crossover probability; Then a crossing site k between positions l and l –1 is generated uniformly at random; Finally, two new strings are created by swapping all the characters between positions k +1and l inclusively. Mutation is performed by randomly altering the value of a bit between 1 and 0 with a mutation probability.

2. 3 A discrete differential evolution algorithm

A discrete DE algorithm is developed and applies to the problem of fuel reloading optimization for the DNRR which loaded with 100 FBs with different fuel burn-up. A parameter vector representing a fuel LP has 100 integer variables with the value in the range from 1 to 100. A NP-size population at generation G in the DE optimization problem consists of NP parameter vectors, each of which is a 100-dimensional parameter vector

$$X_{i,G} = [x_{j,i,G}] \qquad (5)$$

where i = 1, 2, …,NP; j = 1, 2, …, 100.

To initiate DE search process, an initial NP-size population is randomly generated. Each of D variables of NP vectors in the population is randomly assigned by an integer number from 1 to 100.

2.3.1 Mutation

The strategy DE/rand/1/bin, one of the most promising schemes of Storn and Price [19] was used to create the new noisy vector for the next generation (G+1) from the random vectors chosen in the current generation G by the following steps.

An individual of the population is then set as the target vector $X_{i,G}$. A noisy vector is produced by formula (3)

$$V_{i,G+1} = X_{r1,G} + F.(X_{r2,G} - X_{r3,G}) \tag{6}$$

where r_1, r_2, r_3 are randomly chosen among NP populations of generation G. F is the mutation scale factor F that controls the amplification of the differential variation. Variable $v_{j,i,G+1}$ of the noisy vector $V_{i,G+1} = [v_{j,i,G+1}]$ is defined as:

$$v_{j,i,G+1} = x_{j,r1,G} + F.(x_{j,r2,G} - x_{j,r3,G}) \tag{7}$$

2.3.2 Crossover

Crossover operator generates the trial vector $U_{i,G+1} = [u_{j,i,G+1}]$ by the approach given:

$$u_{j,i,G+1} = \begin{cases} v_{j,i,G+1}, & \text{if} \quad rand(j) \le CR \\ x_{j,i,G}, & \text{otherwise} \end{cases} \tag{8}$$

Where *rand*(j) is a random number in the interval [0, 1] are generated and compared with the crossover constant CR, also in [0, 1].

A study on the correlation between *CR* and *F* parameters was proceeded to evaluate their best operation in the discrete DE algorithm for a 10-dimensional parameter vector. Since a large number of calculations, this will be discussed in another report in detail. The obtained control parameters are *CR* = 0.2 and *F* = 0.4.

2.3.3 Selection

The fitness of the adjusted trial vector $U_{i,G+1}$ and target vector $X_{i,G}$ are calculated and compared together. The selection is chosen by the following condition:

$$x_{j,i,G+1} = \begin{cases} u_{j,i,G+1}, & \text{if} \quad Fitness(u_{j,i,G+1}) > Fitness(x_{j,i,G}) \\ x_{j,i,G}, & \text{otherwise} \end{cases} \tag{9}$$

This process is first performed for the first individual of the initial population. To evolve the second individual of the current population for the next generation, it is set as the new target vector and the above process is performed again. The process is repeated until the new population is full of NP new vectors. The search process stops when the termination criterion is met.

Coding procedure for LPs

In order to apply the DE algorithm to the fuel reloading optimization problem, we need a coding procedure to transform a trial vector to an LP and vice versa. In the proposed DE algorithm, the trial vector consists of 100 variables which are 100 integer numbers. The coding procedure developed for GA in section 2.2.1 can be used.

3. RESULTS AND DISCUSSION

In this work, a population of 30 individuals was evolved for 700 generations, so that 21,000 LPs were examined in the application using the DE algorithm. A manner similar to the application using GA, there were totally 42,000 LPs analyzed for the reloading core optimization problem of the DNRR. A binary chromosome has the length of 17, representing a real number between 0 and 1with 5 digits after the decimal point. The weighting factors for the binary mixed integer coded GA are 0.5 and 0.001 of crossover and mutation probability, respectively. These parameters are 0.2 and 0.4 corresponding to the application using the discrete DE algorithm.

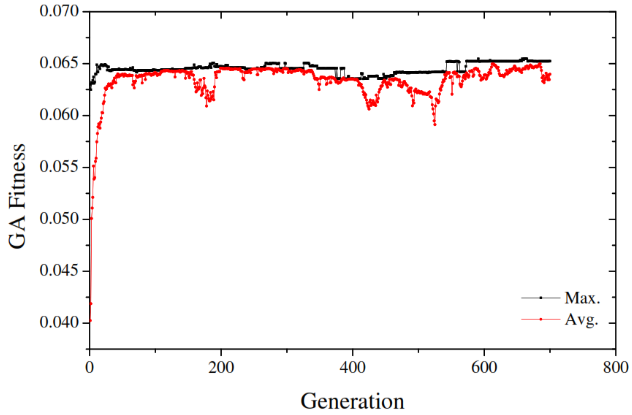


Fig. 2 Change in the fitness by GA versus generation.

Figure 2 shows the change of fitness values after 700 generations of GA application to the optimal problem of the DNRR. It is found that the GA rapidly approaches the fittest solutions after less than 50 generations. The average fitness line is also asymptotic to the maximum line prematurely. It is possible that the elitism strategy which directly transfers the best individuals of the current generation to the next generation has a dramatic impact on GA performance.

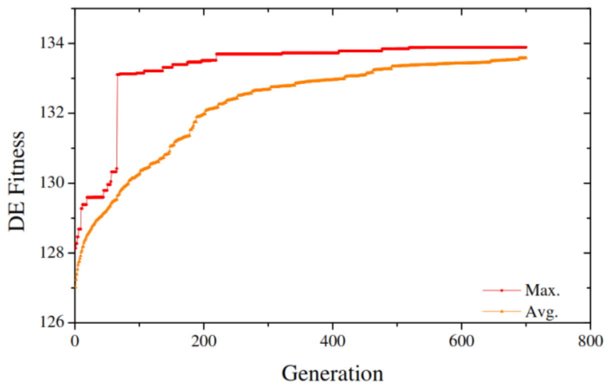
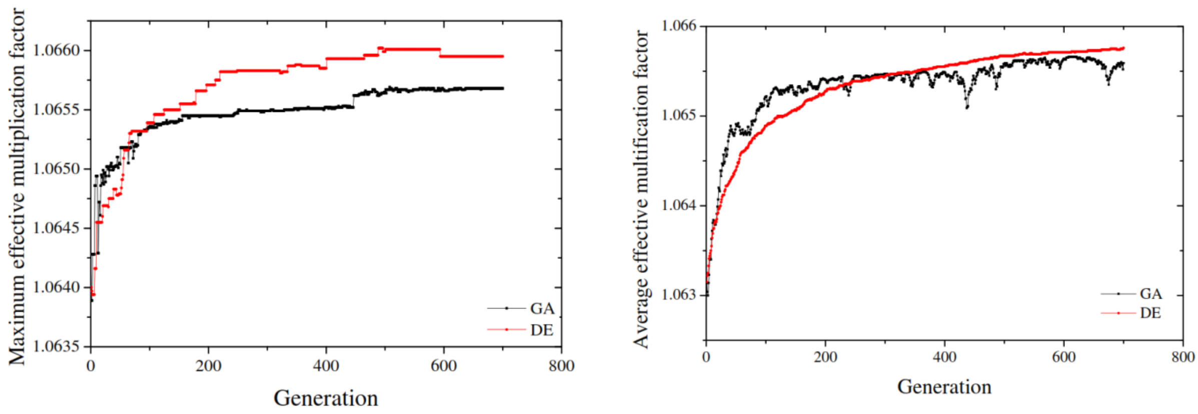


Fig. 3 Change in the fitness by DE versus generation.

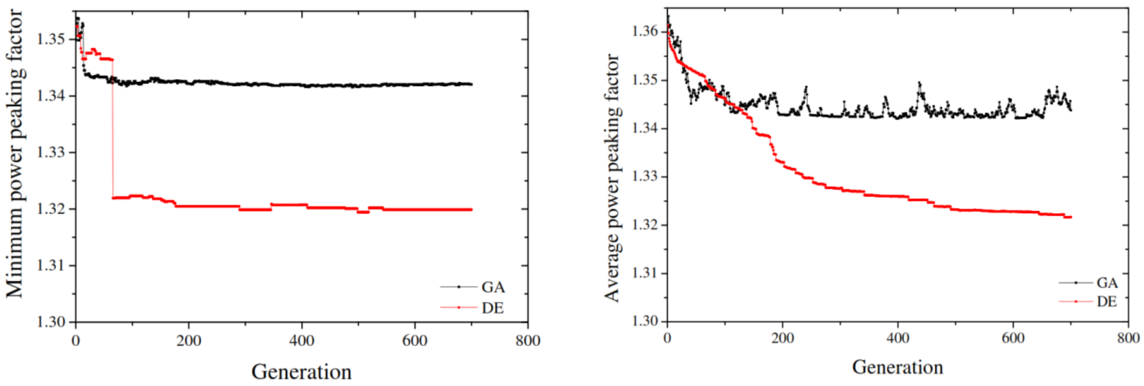
On the other hand, the access of the DE to the fittest solutions is slower but more stable than GA which has shown in Figure 3. This finding agrees well with the “divergence property” of DE discovered by Storn and Price [19] that prevents the search process from advancing slowly in shallow regions of the objective function surface and allows the search process to travel through a narrow valley; hence it is robust in searching for the global solutions.



(a) (b)

Fig. 4 Comparison between GA and DE in the maximum (a) and average (b) effective multiplication factor of population versus generation.

The obtained results on effective multiplication factor and PPF shown in Figures 4-5 indicate the DE to have better performance in finding the optimal solution than the GA. The highest k_{eff} values obtained from GA and DE are 1.06569 and 1.06602, respectively. The DE has the lowest PPF value of 1.31945, as compared with the GA's 1.34157. The average k_{eff} value for the DE is marginally higher while the average PPF for the DE is significantly lower with a value of 1.32164 compared to the GA's 1.34208.



(a) (b)

Fig. 5 Comparison between GA and DE in the minimum (a) and average (b) power peaking factor of population versus generation.

In general, the DE outperforms the GA in terms of the maximum k_{eff}, minimum *PPF*, and the average k_{eff}, *PPF* after 700 generations examine. This indicates that the search process of the proposed discrete DE algorithm in the DNRR problem is more efficient than GA's because DE has a strong possibility to explore search space with better solutions. Moreover, the proposed discrete DE algorithm fulfills the requirement on the convergence properties of a global optimization method like the continuous DE algorithm which has been widely adopted in the field of numerous science problems and nuclear technology applications [20-21].

4. CONCLUSIONS

This paper compared two multi-objective evolutionary algorithms, Differential Evolution and Genetic Algorithms, which were applied to solving the optimization problem of nuclear research reactor fuel reloading. The optimal objectives are both the maximization of the length and the minimization of power peaking factor in a fuel cycle. The algorithms were applied to the DNRR. Comparative analysis between two methods and with the actual work configuration was conducted. The results demonstrate that the performance of the algorithms is satisfactory and the DE is more effective than the GA in the DNRR case.

Acknowledgments

This work was funded by National Foundation for Science and Technology Development (NAFOSTED), Vietnam under grant 103.04-2016.30.

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NGHIÊN CỨU SO SÁNH THUẬT TOÁN TIẾN HOÁ VI PHÂN VÀ THUẬT TOÁN DI TRUYỀN CHO BÀI TOÁN TỐI ƯU HOÁ TÁI NẠP NHIÊN LIỆU LÒ PHẢN ỨNG HẠT NHÂN

Tóm tắt: Tối ưu hoá quản lý nhiên liệu vùng hoạt là một vấn đề rất được quan tâm trong lĩnh vực kỹ thuật hạt nhân. Đây là bài toán tối ưu đa mục tiêu với không gian lời giải lớn, nhiều hàm mục tiêu ràng buộc và không tuyến tính. Bài báo này đề xuất hai cách tiếp cận để giải quyết vấn đề này dựa trên thuật toán tiến hoá vi phân và giải thuật di truyền. Mục tiêu đặt ra là đồng thời tối đa hoá thời gian vận hành và cực tiểu hoá độ bất đồng đều công suất. Thuật toán được áp dụng cho trường hợp của lò phản ứng hạt nhân nghiên cứu Đà Lạt. Các phân tích so sánh

hai phương pháp này với nhau và với cấu hình làm việc thực tế cũng được trình bày. Kết quả chỉ ra rằng cả hai phương pháp đều đáp ứng tốt, trong đó, thuật toán tiến hoá vi phân cho thấy hiệu quả hơn thuật toán di truyền đối với trường hợp lò phản ứng hạt nhân Đà Lạt.
Từ khóa: *Tiến hoá vi phân, giải thuật di truyền, tái nạp nhiên liệu, lò phản ứng hạt nhân Đà Lạt*