EVALUATION SLIP RATIO CORRELATIONS IN TWO-PHASE FLOW
ĐÁNH GIÁ CÁC TƯƠNG QUAN VỀ TỈ SỐ VẬN TỐC TRONG DÒNG HAI PHA

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Abstract: Critical flow is one of the essential parameters in LOCA accident analysis in which pressure different is very high. Void fraction (α), in another term, slip ratio, s, is the key parameter that could effect to critical flow prediction. Henry-Fauske (HF) model is the model for critical flow calculation existing in current computer codes such as MARS, RELAP, TRACE. However, the limitation of this model is slip ratio s=1. By modified the slip ratio correlation, the paper focuses on evaluating the HF model. Among the chosen correlations for slip ratio, Smith correlation is the best option for this purpose. The results in our paper showed that while original Smith correlation with k=0.4 is suggested for horizontal tests, the modified one with k=0.2 could be applied for vertical tests.

Key words: *Void fraction, slip ratio, critical flow.*

Tóm tắt: *Vận tốc cực tới hạn là một trong những thông số quan trọng trong phân tích an toàn cho các sự cố mất chất tải nhiệt của các lò phản ứng hạt nhân mà trong đó có độ chênh áp cao. Độ rỗng (α) hay theo một cách diễn đạt khác, tỉ số vận tốc,s, chính là thông số chính có thể tác động đến dự đoán vận tốc tới hạn. Mô hình Hery-Fausle (HF) là mô hình tính toán vận tốc tới hạn hiện tại trong các phần mềm tính toán như MARS, RELAP hay TRACE. Tuy nhiên nhược điểm của mô hình này là s=1. Thông qua việc thay đổi hệ số s, bài báo này tập trung vào việc đánh giá mô hình HF. Trong tổng số các hàm tương quan cho tỉ số vận tốc s, hàm tương quan Smith là lựa chọn tốt nhất để đánh giá vận tốc tới hạn. Các kết quả trong báo cáo này chỉ ra rằng, trong khi hàm tương quan của Smith với hệ số k=0.4 là lựa chọn cho các hệ thí nghiệm nằm ngang, hàm tương quan thay đổi của Smith với hệ số k=0.2 có thể áp dụng tốt đối với các hệ thực nghiệm thẳng đứng.*

Từ khóa: *Void fraction, slip ratio, critical flow.*

1. INTRODUCTION

Critical flow phenomenon takes place when liquid, gas or mixture leaks from a system at high pressure to the ambient at lower pressure through a break such as in a break of safety valves or safety injection lines during a loss of coolant accident (LOCA). Understanding of break flow and its modeling are important things in a LOCA scenario. When the system approaches the critical flow condition, the discharge flow through the broken exit could reach a maximum value, and the flow rate becomes independent from the downstream pressure. Henry-Fauske (HF) and Trapp-Ransom (TR) are two leading applicable models for critical flow calculation in the current safety analysis codes among many models. However, as the application of these models is different, they should be used carefully while doing the calculation. HF model is only applicable for one component steam-water system. Furthermore, based on the assumption of the HF model, it could be applied to a well-mixed condition, thermal equilibrium. TR model, however, is applied to the two-component system, air-water. Together with this, to predict the critical mass flow, the HF model uses the upstream conditions, while the TR model bases on the throat conditions. These models used a void fraction to predict the critical flow. Void fraction, defining as the fractional occupied area of the gas phase, is one important parameter for two-phase flow, based on which the component pressures, flow rate, and heat transfer are determined. Because the gas phase normally moves faster than that of the liquid phase, the void fraction could not be directly calculated from the mass flow rates of each phase separately. Therefore, void fraction depends on the phase velocity ratio, so-called slip ratio. Slip ratio can be influenced by many variables, such as mixture quality, temperature and pressure, the direction of flow, circulation mode, wall friction Fauske [3], and the system geometry Kim [17]. Among those affected parameters, Kim [17] has suggested that the effect of diameter was the most important one influencing to the critical flow. His explanation for this effect was related to slip ratio.

This paper first reproduced the HF numerically. Our reproduced model for HF showed a good agreement in comparing with the selected database. Furthermore, based on Kim' suggestion, various slip ratio correlations were taken into account for evaluating the critical flow rate. However, the model of HF has only considered the slip ratio is unity. Therefore, our focus is evaluated HF model with different slip ratio correlation. Our result showed that Smith [14] correlation was the best candidate for predicting the critical flow. Based on his original k =0.4 value, the author modified this value. It could conclude that among chosen slip correlations, Smith correlation gave a good agreement while varying the k parameter for both horizontal and vertical tests. While Original Smith correlation could be used for the horizontal test, the modified k=0.2 for this correlation could be applied for a vertical one.

2. HENRY-FAUSKE CORRELATION

2. 1. Henry-Fauske correlation

Henry-Fauske [1] suggested the model for a non-equilibrium model, using two continuity and one momentum equations of the single component flow (water–vapor system) without considering the wall shear stress and heat exchange. This model also developed using the following approximations:

- Same phase velocity (k=1)
- No mass transfer in the expansion
- Being in thermal equilibrium
- Isentropic expansion (s_g=s_l)
- The same liquid temperature.
- Polytropic expansion of vapor at the exit (as the ideal gas).
- Critical mass flow rate reaches a maximum value concerning the throat pressure, dG/dp_t =0.

Based on these approximations, the flow rate can be determined as follows:

$$\mathrm{G_{HF}}^2 = \left[\frac{\mathrm{x_0 v_{gt}}}{\mathrm{nP}} + (\mathrm{v_{gt}} - \mathrm{v_{f0}})\left\{\frac{(1-\mathrm{x_0})\mathrm{N}}{\mathrm{s_{gt}}-\mathrm{s_{ft}}}\frac{\mathrm{ds_{ft}}}{\mathrm{dP}} - \frac{\mathrm{x_0 c_{Pg}}\left(\frac{1}{\mathrm{n}}-\frac{1}{\gamma}\right)}{\mathrm{P}(\mathrm{s_{g0}}-\mathrm{s_{f0}})}\right\}\right]^{-1} \tag{2.1.1}$$

Integrating the momentum equation from the stagnant to the throat locations:

$$(1-\mathrm{x_0})\mathrm{v_{f0}}(\mathrm{P_0}-\mathrm{P_t}) + \frac{\mathrm{x_0}\gamma}{\gamma-1}\left(\mathrm{P_0 v_{g0}} - \mathrm{P_t v_{gt}}\right) = \frac{\left[(1-\mathrm{x_0})\mathrm{v_{f0}} + \mathrm{x_0 v_{gt}}\right]^2}{2} G_{\mathrm{HF}}{}^2 \tag{2.1.2}$$

Substitution eq. (2.1.1) into the eq. (2.1.2) and rearrange Eq. (2.1.2) the compact form can be obtained:

$$\eta = \left[\frac{\frac{(1-\alpha_0)}{\alpha_0}(1-\eta) + \frac{\gamma}{\gamma-1}}{\frac{1}{2\beta{\alpha_t}^2} + \frac{\gamma}{\gamma-1}}\right]^{\frac{\gamma}{\gamma-1}} \tag{2.1.3}$$

where

$$\eta = \frac{\mathrm{P_t}}{\mathrm{P_0}} \tag{2.1.4}$$

$$\beta = \frac{1}{\eta} + \left(1 - \frac{v_{f0}}{v_{gt}}\right)\left(\frac{(1-x_0)NP_t}{x_0\left(s_{gt}-s_{ft}\right)}\frac{ds_{ft}}{dP}\right) - \frac{c_{pg}\left(\frac{1}{n}-\frac{1}{\gamma}\right)}{\left(s_{g0}-s_{f0}\right)} \tag{2.1.5}$$

and

$$\alpha_0 = \frac{x_0 v_{g0}}{(1-x_0)v_{f0} + x_0 v_{g0}}, \alpha_t = \frac{x_0 v_{gt}}{(1-x_0)v_{f0} + x_0 v_{gt}} \text{ and } v_{gt} = v_{g0}(\eta)^{-\frac{1}{\gamma}}$$

For given stagnant conditions of P_0 and x_0, by iteration until the η values in two Eq. (2.1.3) and Eq. (2.1.4) are converged, the critical pressure, P_t, can be obtained. The critical mass flow rate finally can be calculated.

2.2 Reproducing and comparison of HF model

One of the author's difficulty is that MARS code is not commercial software. It is only allowable for Korean students to use its source code. Therefore, to modify the correlations in the HF model, the author has numerically reproduced this model based on the main equations given in the original papers [1].

Comparisons have been made for one component steam-water. The critical mass flux data versus stagnation quality are taken from Henry and Fauske [1] at different pressures. Figs 1, 2 and 3 show the comparisons between reproducing HF model and the experimental data [1]. It could be concluded that the HF reproducing model give similar results to the original one.

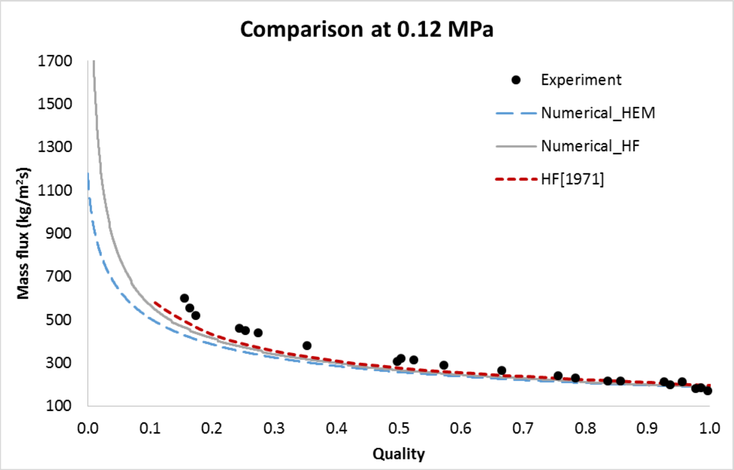

Fig. 1. Comparison of reproducing HF model with experimental data at 0.12MPa (17.6 psi) [1].

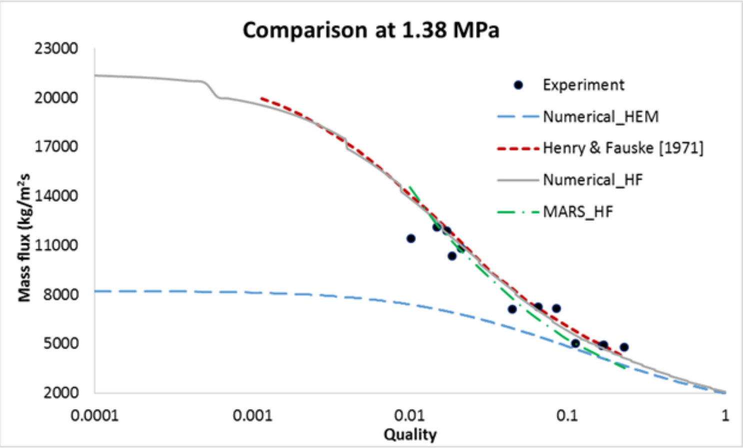

Fig. 2. Comparison of reproducing HF model with experimental data at 1.38MPa (200 psi) [1].

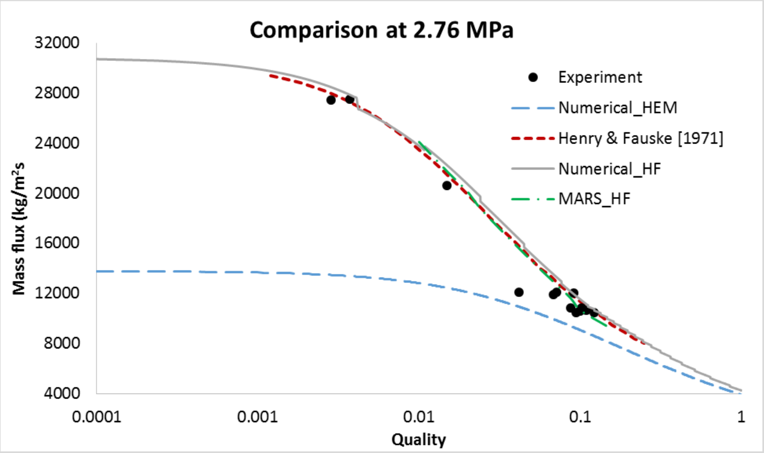

Fig. 3. Comparison of reproducing HF model with experimental data at 2.76 MPa (400 psi) [1].

From Figs. 1, 2 and 3, we can see that the HF model tends to be better at high pressure, and its result is under-predicted the data at low pressure. It means that this model in the current MARS code could be used for high-pressure systems.

3. SLIP RATIO CORRELATIONS

The estimation of hydrostatic, acceleration, and friction pressure drops, and critical mass flow rate in the two-phase mixture is based on the knowledge of the void fraction, the relative volumes of gas and liquid phases at a given location. A huge number of empirical correlations and theoretical models for void fraction have been proposed in the past [9, 10]. The theoretical ones are mainly based on simplifying assumptions [1,2,4], respect to the flow regime, and therefore the results cannot be of general applicability to two-phase phenomena such as boiling process including several different flow regimes, from bubbly to separate flow. Other models are developed based on a limited range of experimental conditions such as pressure, temperature or quality [14,15,16]. The void fraction depends on the phase velocity ratio, slip ratio in the general form [9,10] as a function of quality (x), density (ρ) and viscosity (μ):

$$\alpha = \frac{1}{1 + A\left(\frac{1-x}{x}\right)^{b}\left(\frac{\rho_g}{\rho_f}\right)^{c}\left(\frac{\mu_f}{\mu_g}\right)^{d}} \tag{3.1}$$

In reality, only several types of reactors for gas/liquid exist. Moreover, one could be noted that the contribution of the viscosity component as shown in Table 1, $\left(\frac{\mu_f}{\mu_g}\right)^{d}$, is dominant. Therefore, this parameter could be negligible. Void fraction correlations then could be reduced as follows:

$$\alpha = \frac{1}{1 + A\left(\frac{1-x}{x}\right)^{b}\left(\frac{\rho_g}{\rho_f}\right)^{c}} \quad \text{or} \quad \alpha = \frac{1}{1 + s\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_f}\right)} \tag{3.2}$$

Where the slip ratio, s, is determined as follows:

$$s = A\left(\frac{1-x}{x}\right)^{b-1}\left(\frac{\rho_g}{\rho_f}\right)^{c-1} \tag{3.3}$$

Select the void fraction correlations having the form in Eq. (3.2), the author has compared them with the available experimental data (Table 2).

Void fraction correlations related to slip ratio are listed in Table 1:

Table 1. Some void fraction correlations

Correlation	**A**	**b**	**c**	**d**
HEM	1	1	1	0
Fauske (1961)	1	1	0.5	0
Zivi (1964)	1	1	0.67	0
Smith (1969)	$k + (1-k)\sqrt{\dfrac{\dfrac{\rho_L}{\rho_G} + k\left(\dfrac{1-x}{x}\right)}{1 + k\left(\dfrac{1-x}{x}\right)}}$	1	1	0
Chisholm (1973)	$\sqrt{1 + x\left(\dfrac{\rho_L}{\rho_G} - 1\right)}$	1	1	0
Spendding & Chen (1984)	2.22	0.65	0.65	0
Hamersma & Hart (1987)	0.26	0.67	0.33	0
Tuner & Wallis (1965)	1	0.72	0.4	**0.08**
Lockhart & Martinelli (1949)	0.28	0.64	0.36	**0.07**
Thom (1964)	1	1	0.89	**0.18**
Baroczy (1966)	1	0.74	0.65	**0.13**
Premoli et al. (1970)	A_{PRM}	*1*	*1*	*0*
Madsen (1975)	*1*	*M*	*-0.5*	*0*
Chen (1986)	*0.18*	*0.6*	*0.33*	*0.07*
Petalaz and Aziz (1997)	A_{PA}	*-0.2*	*-0.126*	*0*

where: Re and We are Reynolds and Weber numbers, D is the equivalent diameter

$$A_{PRM} = 1 + F_1\left\{\frac{y}{1 + yF_2} - yF_2\right\}, F_1 = 1.578(Re)_f^{-0.19}\left(\frac{\rho_g}{\rho_f}\right)^{-0.22}, F_2 = 0.0273We_f\,(Re)_f^{-0.51},$$

$$y = \left(\frac{1-x}{x}\right)^{-1}\left(\frac{\rho_g}{\rho_f}\right)^{-1}, (Re)_f = \frac{GD}{\mu_f}, We_f = \frac{G^2D}{\sigma\rho_f}, \text{G is the mass flux}$$

$$M = 1{+}\log\left(\frac{\rho_f}{\rho_g}\right)/\log\left(\frac{1-x}{x}\right) \quad , \quad A_{PA} = 0.735(\mu_f)^2(U_{SG})^2/\sigma^2, \ \sigma \text{ is surface tension}$$

Table 2. References related void fraction data measurement

References	**Diameter [mm]**	**Working fluid**	**Geometry**	**Pressure [psi]**
Marchettere[1956], Cook [1956]	127	Steam-Water	Rectangular, vertical	114 to 600
Haywood [1961]	12.7-38.1	Steam-Water	Pipe, horizontal	250 to 2100

Kim [17] has currently suggested that the slip ratio is the main parameter that could affect the diameter effect. In the past, some experimental work showed that critical mass flow rate increased while reducing the diameter of the throat (Fauske [3], Sozzi and Sutherland [19], Chun and Park [20]). This could be explained mainly based on diameter effect. The increase of vaporization may be higher at the choking place with the decrease in sub-cooling upstream condition at very low sub-cooling temperature nozzle. This means that the slip ratio may increase for low sub-cooling upstream conditions. Several slip ratio correlations were reviewed before taking them into account their critical flow predictions.

By comparing with the experimental data, void fraction correlations listed in Table 1 are evaluated. The 6 secleted correlations then rewiten in the slip ratio form as shown in Table 3.

Those slip ratio correlations will be evaluated more detail using different pressures for both horizontal [13] and vertical [11,12] tests.

Table 3. Slip ratio correlations

Correlation	**Slip ratio, s**
HEM	1
Fauske (1961)	$\left(\frac{\rho_g}{\rho_f}\right)^{-1/2}$
Zivi (1964)	$\left(\frac{\rho_g}{\rho_f}\right)^{-2/3}$
Smith (1969)	$k + (1-k)\sqrt{\frac{\frac{\rho_L}{\rho_G} + \frac{k(1-x)}{x}}{1 + k\left(\frac{1-x}{x}\right)}}$
Spedding & Chen (1984)	$2.22\left(\frac{1-x}{x}\right)^{-0.35}\left(\frac{\rho_g}{\rho_f}\right)^{-0.35}$
Hamersma & Hart (1987)	$0.26\left(\frac{1-x}{x}\right)^{-1/3}\left(\frac{\rho_g}{\rho_f}\right)^{-2/3}$

3.1 Evaluations using horizontal experimental data

Based on the comparison of chosen correlations with Haywood data for a horizontal test at high pressures from 1.72 to 14.5 MPa as shown in Figs. 4, 5, 6 and 7, we can conclude that Smith [14] correlation with his recommended k = 0.4 gives the best predictions.

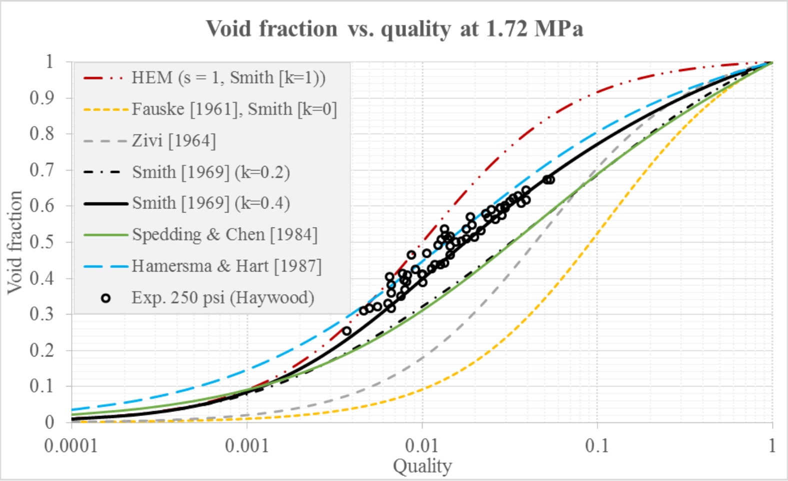

Fig. 4 Void fraction vs. quality in comparing with the Haywood data at 1.72 MPa (250 psi) [13].

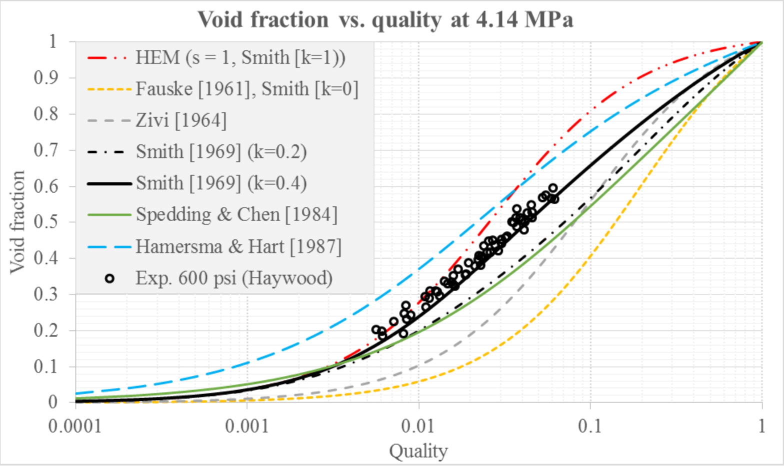

Fig. 5 Void fraction vs. quality in comparing with the Haywood data at 4.14 MPa (600 psi) [13].

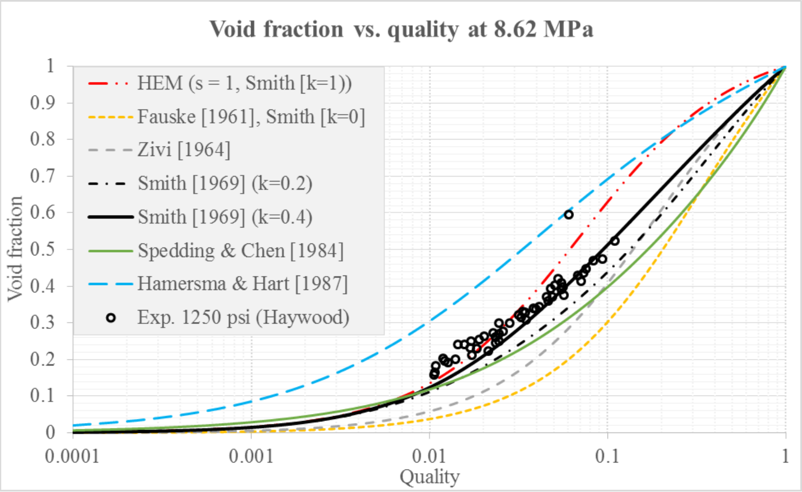

Fig. 6 Void fraction vs. quality in comparing with the Haywood data at 8.62 MPa (1250 psi) [13].

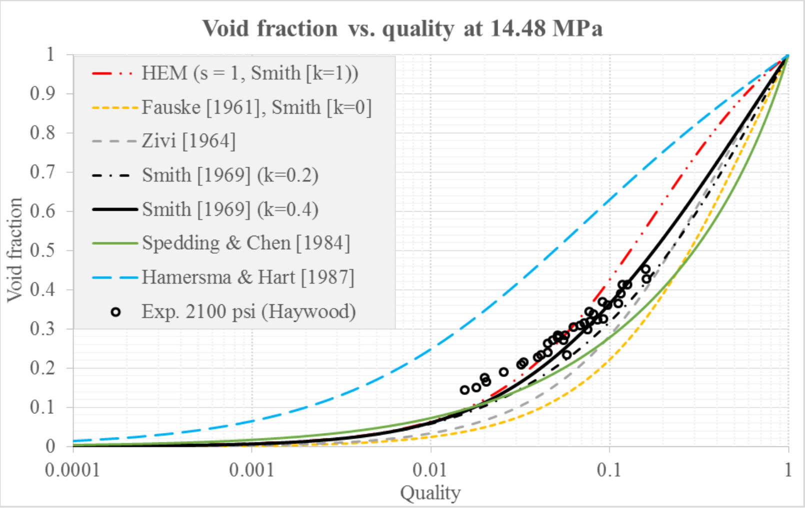

Fig. 7 Void fraction vs. quality in comparing with the Haywood data at 14.48 MPa (2100 psi) [13].

3.2 Evaluations using vertical experimental data

Marchettere [11] and Cook [12] performed experiments using the same vertical test facility but different pressures from 0.69 to 4.14 MPa (100 to 600 psi). The predictions of chosen correlations are compered with their data as follows:

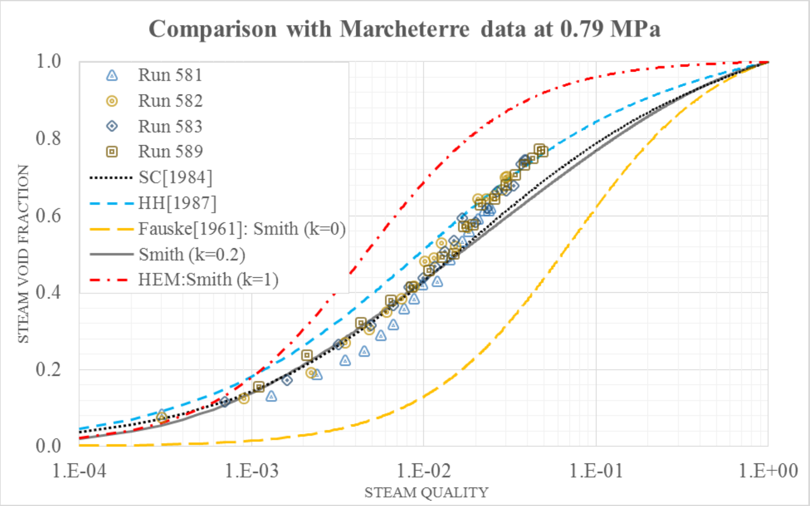

Fig. 8 Void fraction vs. quality in comparing with the experimental data at 0.79 MPa (114.5 psi) [11].

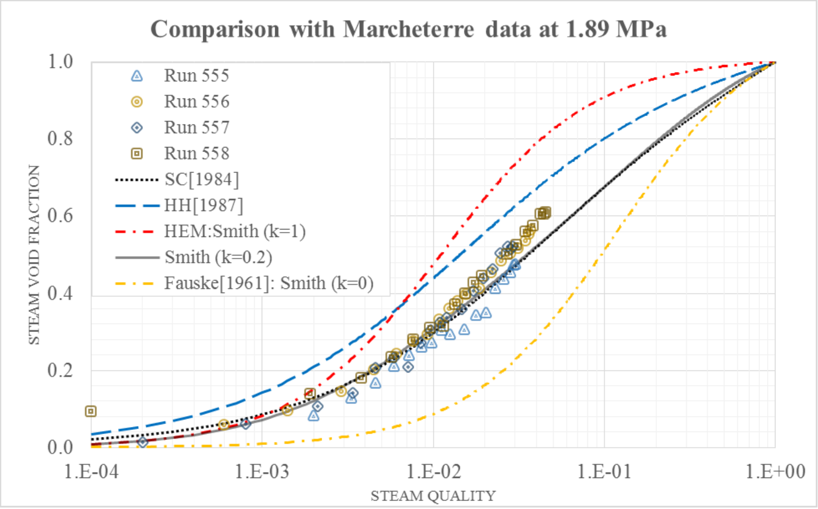

Fig. 9 Void fraction vs. quality in comparing with the Experimental data at 1.89 MPa (274.3 psi) [11].

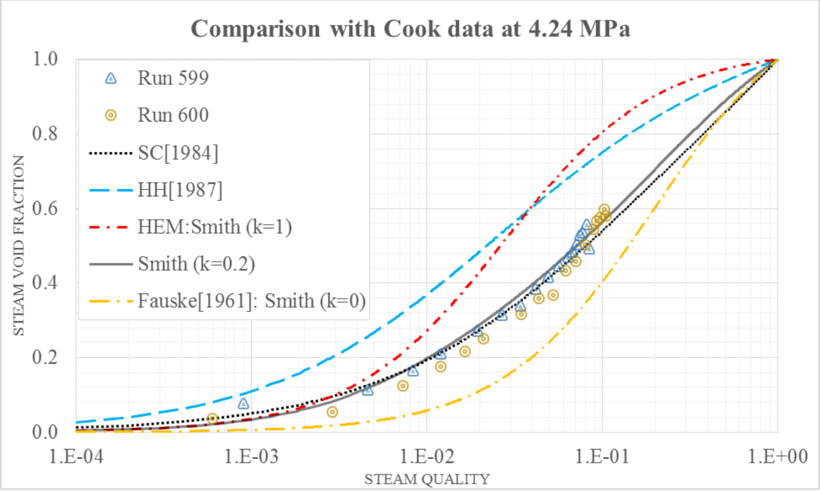

Fig. 10 Void fraction vs. quality in comparing with the experimental data at 4.24 MPa (614.4 psi) [12].

The comparison between the chosen correlations results and the experimental data from Marchettere [11] and Cook [12 shows that, the modified Smith [14] correlation with k = 0.2, as well as Speeding and Chen [16] one, gives the best predictions as shown in Figs. 8, 9 and 10.

From these above comparisons, it could be concluded that while the original Smith correlation with k=0.4 seems to be the best one for the horizontal test, its modification with k=0.2 could be the best correlation for the vertical test.

3.3 Evaluation of choosing Smith correlation for critical mass flux prediction

Modifying the original slip ratio, s =1, in HF model by using these chosen slip correlations to predict the critical mass flux, the result can be seen in Figs 11, 12 and 13. This process is evaluated using 0.12, 1.38 and 2.76 MPa data. The critical mass flux results of original HF is plotted in black continuous line, and its modified one using Smith correlation with k = 0.4 is in red long dash line. The compared resutls showed that while the predictions of mass flux at high quality are quite the same, they become different at low quality. The results at low pressure (Fig. 11) showed a similar result for all correlations at a quality higher than 0.1. In comparing with the original HF model (s=1), modified HF model using Smith correlation gives a better prediction in case of high pressures (Figs. 12 and 13). The modified results using Spedding & Chen and Hamersma & Hart show bad prediction at very low quality (less than 10^{-3}). We could see clearly that among these slip correlations, Smith correlation with k=0.4 is the best option for critical mass flux prediction.

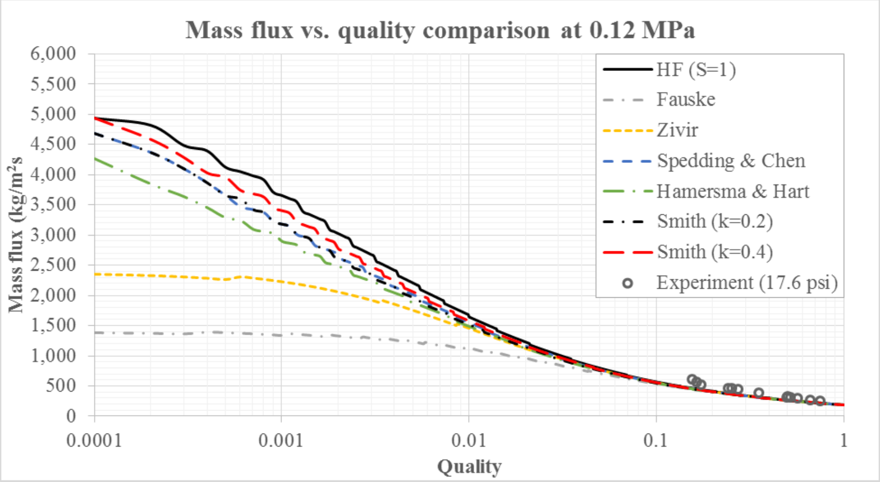

Fig. 11 Evaluation of critical mass flux for the HF model using the experimental data at 0.12 MPa [1]

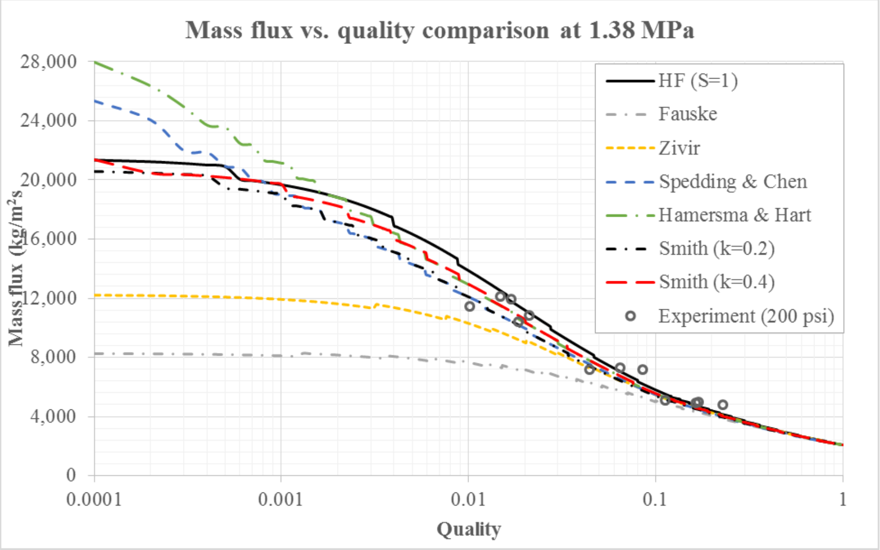

Fig. 12 Evaluation of critical mass flux for the HF model using the experimental data at 1.38 MPa [1]

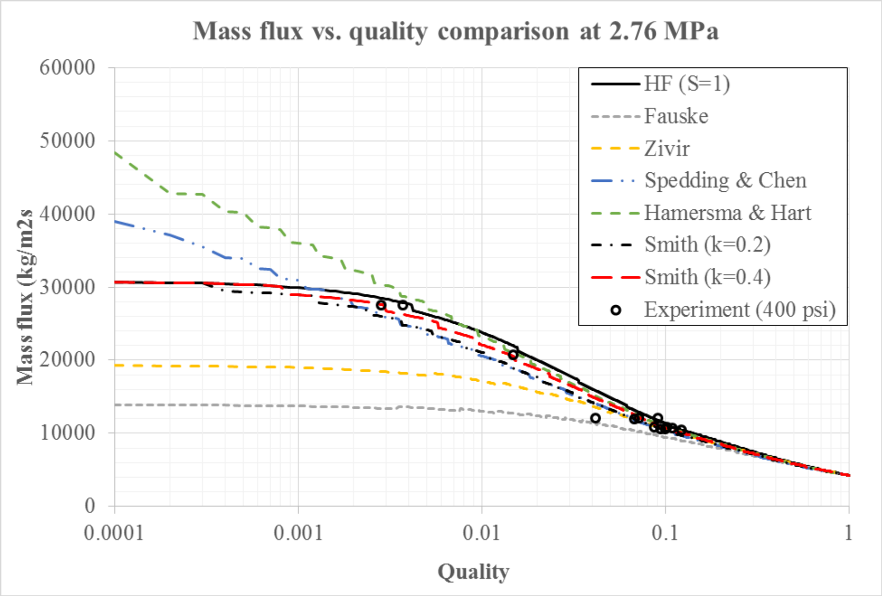

Fig. 13 Evaluation critical mass flux for the HF model using the experimental data at 2.76 MPa [1]

4. CONCLUSION AND FUTURE WORK

Due to the limitation in software handling, the reproducing model of HF has been successfully evaluated. The reproducing model showed similar results with that calculated by using the original HF model. Slip ratio correlations, which correlate with critical mass flux predictions, were chosen and evaluated using both horizontal and vertical tests. From this work, we could conclude that the original Smith correlation with k=0.4 is the best choice for horizontal tests, while the modified one with k=0.2 is applicable for the vertical test. Further data evaluation is needed for a wider range of pressure for both horizontal and vertical tests to get a clear picture of the best option for critical flow rate prediction.

NOTATION

c	Sound velocity (m/s)
C	Vitural mass (kg)
D	Diameter (m)
G	Mass flux (kg/m^2/s)
h	Enthalpy (J/kg)
k	
L	Length (m)
N	
s	Slip ratio, (u_g/u_f)
S	Entropy (J/K)
x	Quality
P	Presssure (MPa)
u_f	Liquid velocity (m/s)
u_g	Gas velocity (m/s)
v_f	Specific volume of liquid (m^3/kg)
v_g	Specific volume of gas (m^3/kg)
We	Weber number
Re	Reynolds number

Superscripts

0	Stagnant location
t	Throat location
g	Gas component
f	Fluid component

Greeks

α	Void fraction
γ	Isentropic exponent
λ	The root of characteristic equation
η	Critical pressure ratio
ρ_f	Liquid density (kg/m^3)
ρ_g	Gas density (kg/m^3)
μ_f	Liquid viscosity (Ns/m^2)
μ_g	Gas viscosity (Ns/m^2)

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