STUDY OF MEDIUM EFFECT BY ^{12}C - ^{12}C ELASTIC SCATTERING ANALYSIS AT LOW ENERGIES

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Abstract: The medium effect is investigated by studying the ^{12}C-^{12}C elastic scattering in the framework of the double folding model using the realistic CDM3Y3 effective nucleon-nucleon (NN) interactions and the wave functions of interacting nuclei. Both frozen and adiabatic density approximation are used for this purpose. The Hartree-Fock (HF) type potentials corresponding to the two approximations are tested with the elastic scattering data of ^{12}C + ^{12}C system in the low energy range based on the Optical Model (OM) analysis. The obtained results propose that the adiabatic density approximation is more reasonable than the frozen density approximation in describing the overlapping density for ^{12}C + ^{12}C system at low energy region.

Key words: *Elastic scattering, Optical Model, adiabatic approximation*

I. INTRODUCTION

The ^{12}C + ^{12}C reaction is an interesting topic that has attracted many researches in both the experimental and theoretical fields over four decades. There are many studies in the experimental field to measure the data over a wide range of energies [1-7], which are analysed theoretically by employing both the phenomenological and microscopic potentials [8-11]. The experimental data of angular distributions, typically for the region of energies above 10 MeV per nucleon, have been studied and explained unambiguously using potential of which the real part has deep strength [8]. However, in the energy region below 10 MeV per nucleon, the experimental data have not been analysed clearly and the question of potential family that is typical for this energy range still remains ambiguities. In addition, the ^{12}C + ^{12}C fusion process at energies near and below Coulomb barrier plays an important role in studying the Carbon-burning process, known as a chain to yield heavier elements in stars and has attracted a large number of interest in nuclear astrophysics since 1960 [6-7,11,12-16]. From the survey, it should be interested to study the ^{12}C + ^{12}C microscopic interaction potential at this energy region.

When studying the microscopic potential, it is always of interest to determine this potential starting from the nucleon degrees of freedom within the nuclear mean field formed during the di-nuclear collision. And the folding model, which is obtained by averaging an appropriate nucleon-nucleon (NN) interaction over the matter distributions [8,17-19], is an appropriate approach for this purpose. Many analyses focus on the study of the effective interaction and nuclear distribution, which are known as the important inputs of folding model [8,17-19]. Besides, the medium effect, which is defined as the nuclear environment around the interaction between two nucleons, is an physical object needed to investigate. In this paper, two kinds of frozen and adiabatic approximations are employed to investigate the medium effect during the collision process of two interacting nuclei at low energies. The microscopic potentials corresponding to two medium effect approximations is constructed in the framework of double folding model using the appropriate effective NN interaction and the nuclear densities. Especially, the CDM3Y3 version of the realistic density dependent effective NN interactions [19] and the two-parameter Fermi distributions [20] are taken into account to calculate the Hartree-Fock potential type. The obtained potentials are tested by analysing the angular distributions of ^{12}C-^{12}C elastic scattering at low energy range (below10 MeV per nucleon).

In Sec. II, we discuss the theory of Optical Model Potential (OMP) and Double Folding Model (DFM). Some obtained results of the potentials and angular distribution analyses are given in Sec. III. We summarize and conclude in Sec. IV.

II. FORMALISM

II.1. Optical model

In general, the quantum scattering of incident particles from a target is described by the differential cross section [21]

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \tag{1}$$

The value of scattering amplitude $f(\theta)$ is determined according to the optical model by solving the Schrodinger equation for elastic nucleus - nucleus scattering

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + U(r)\right]\psi = E\psi, \tag{2}$$

in here, $\mu = \frac{m_A m_a}{m_A + m_a}$ is the reduced mass (A, a are the labels of projectile and target nuclei, respectively), U(r) is the complex potential given in the form

$$U(r,E) = V_R\,(r,E) + iW_I\,(r,E) + V_C\,(r) \tag{3}$$

V_C is the Coulomb potential. U(r,E) includes $V_R(r,E)$ and $W_I(r,E)$ which correspond to the real and imaginary components. We calculate V_R (r,E) by applying the folding model and W_I (r,E) part with using the phenomenological Woods-Saxon shape given in form

$$W_I(\mathrm{r},\mathrm{E}) = \frac{W_0(\mathrm{E})}{1 + \exp(\frac{r - R_I}{a_I})} \tag{4}$$

II.2. Microscopic potential

II.2.1 Double folding potential

As an important input for optical model calculations, the nuclear potential is performed microscopically in the framework of double folding model and started from nucleon degrees of freedom. From the physical point of view, the nuclear potential V_R is assumed as the sum of the effective NN interactions v_{ij} and given by formula below

$$V_R = \sum_{i\in a,\, j\in A} \left[\langle ij|v_D|ij\rangle + \langle ij|v_{EX}|ji\rangle\right] = V_{aA}^{D} + V_{aA}^{EX}. \tag{5}$$

With $V_{aA}^{D}\,\mathrm{and}\,V_{aA}^{EX}$ are direct and exchange terms, respectively

$$V_{aA}^{D}(\vec{R}) = \int \rho_a(\vec{r}_a)\,\rho_A(\vec{r}_A)\,v_D(s)\,d^3r_a\,d^3r_A, \tag{6}$$

$$V_{aA}^{EX}(\vec{R}) = \int \rho_a(\vec{r}_a, \vec{r}_a + \vec{s})\,\rho_A(\vec{r}_A, \vec{r}_A - \vec{s})\,v_{EX}(s)\exp\left(\frac{i\vec{k}(\vec{R})\cdot\vec{s}}{M}\right) d^3r_a\,d^3r_A. \tag{7}$$

In here, $\vec{s} = \vec{r}_A - \vec{r}_a + \vec{R}$ is the relative distance between the projectile and target. r_a, r_A are the nucleon coordinates inside the A target and a projectile, correspondingly. E is the energy in the center of mass system. And the v_D and v_{EX} are the direct and exchange terms of v_{NN} interaction, respectively.

Two important inputs for the calculation of the Hartree-Fock potential are the nuclear densities and the effective NN interactions. The two-parameter Fermi distribution is used for describing the ground-state nuclear density

$$\rho(r) = \rho_0 \left\{1 + \exp\left(\frac{r-c}{a}\right)\right\}^{-1}, \tag{8}$$

with the set of parameters (ρ_0 [illegible]) chosen to reproduce correctly the empirical nuclear root-mean-square (r.m.s) radius which is extracted from elastic electron scattering [20]. And the realistic CDM3Yn versions of density dependent NN interactions (v_D or v_{EX}) are defined as

$$v_{D(EX)} = F(\rho)\, v_{D(EX)}(s). \tag{9}$$

In here, the radial dependences of M3Y interactions are defined as a sum of three Yukawa functions with parameters adjusted to satisfy the saturation properties of nuclear matter [22,19]

$$v_D(s) = 11061.6 \frac{\exp(-4s)}{4s} - 2537.5 \frac{\exp(-2.5s)}{2.5s}, \tag{10}$$

$$v_{EX}(s) = -1524.25 \frac{\exp(-4s)}{4s} - 518.75 \frac{\exp(-2.5s)}{2.5s} - 7.8474 \frac{\exp(-0.7072s)}{0.7072s}. \tag{11}$$

Besides, $F(\rho)$ is known as the function which emphasizes that the NN interaction between two nucleons inside the nuclei is not equivalent to that in the free space and it should be strongly taken into account the effects of surrounding nucleons. It is so called for the medium effect which is described in detail later and given in form

$$F(\rho) = C[1 + \alpha \exp(-\beta\rho) + \gamma\rho] + C'[\alpha' \exp(-\beta'\rho) - 1]. \tag{12}$$

These parameters (C, α, β, γ) are chosen to yield K values ranging from 188 to 252 MeV while still reproduce correctly the saturation values of nuclear matter such as density and binding energy. Especially, CDM3Y3 version of the effective NN interaction with parameters ($C = 0.2985, \alpha = 3.4528, \beta = 2.6388\, fm^3, \gamma = -1.5\, fm^3, C' = 0.38, \alpha' = 1, \beta' = 4.484$) yields a nuclear incompressibility K = 218 MeV [18].

II.2.2. Treatment for medium effect

II.2.2.1 Frozen density approximation

In the present double folding calculations, there are several descriptions for overlapping densities, which are known as the nuclear environment around the NN interaction between two nucleons. The widely used approximation in double folding calculations is defined as the sum of the local densities of projectile and target corresponding to the individual positions of interacting nucleons

$$\rho = \rho_a(\vec{r}_a) + \rho_A(\vec{r}_A). \tag{13}$$

This approximation, which is called the "frozen density approximation" (FDA), is successful for describing the overlapping density in many cases of di-nuclei interactions at the intermediate and high energy regions [8,17-19].

II.2.2.2. Adiabatic density approximation

In the region of low energies which the approaching speed of two interacting nuclei is low in comparison with nucleons' speed in nuclei, after collision the nuclei try to penetrate into the other for composing the compound nucleus (CN) while nucleons inside the CN have enough time to change their single energy levels to make nuclear density vary gradually (corresponding to the adiabatic condition, so called adiabatic density approximation; ADA). To describe the circumstance, the densities of target and projectile nuclei are assumed to change during approaching process such that the overlapping density at the central point dose not exceed the nuclear compound density. Especially for ^{12}C identical system, the compound nucleus is ^{24}Mg nucleus. The parameters of the two-parameter Fermi distributions are changed to achieve this in the way [23]

$$C_{dau}(R)=\begin{cases} C_{dau}(0)\exp\left\{\ln\left(\frac{C_{par}}{C_{dau}(0)}\right)\left(1-\frac{R}{R_{cut}}\right)^{2}\right\} & R\le R_{cut} \\ C_{dau}(0) & R>R_{cut} \end{cases}. \tag{14}$$

The sub-label "dau" (or "par") represents for daughter (or compound) nuclei. The formula is similar to the *a* parameter. And the normalization condition is used for calculating the ρ_0 values as follows

$$4\pi\int_{0}^{\infty}\rho_{A}(r)r^{2}dr=A. \tag{15}$$

III. RESULTS AND DISCUSSIONS

In the Fig. 1, we present the results of overlapping densities with both frozen and adiabatic density approximations. In the case of ADA, Rcut is chosen at which the reorganization of the central part of the compound nucleus into two central densities of individual daughters starts to occur. In the physical point of view, the separation between two daughter nuclei, Rcut, is roundly equal to 3.8 fm [23]. One can see in the Fig. 1, in the FDA case, the overlapping density at the central point reaches to twice the saturation density of ^{12}C individual daughter nucleus (~0.388 fm^{-3}) while that is equivalent to the ^{24}Mg compound nucleus density (~ 0.167 fm^{-3}) corresponding to ADA calculations. The results, shown in Fig. 1, point out that in the adiabatic regime, the overlapping density changes gradually during the collision process and starts to dilute at the contact point in order to merge easily two nuclei into each other. In contrast, the overlapping density, in the case of FDA, alters quickly with a tendency to make the compound nucleus more tightly, which makes two daughter nuclei difficult to join together at low energies.

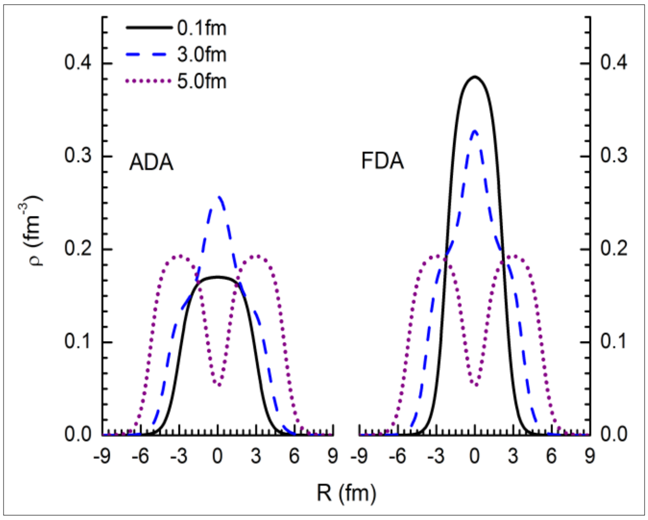

Fig. 1. The overlapping density within both frozen and adiabatic density approximations as a function of the relative distance between two daughter nuclei.

The real parts of Hartree-Fock type potentials are constructed in the framework of double folding with two types of frozen and adiabatic density approximations which are so called FDA and ADA potentials, respectively. As illustrated in the Fig.2, the dashed line presents for FDA potential and remain is shown by the solid line. The results show that the ADA potential drops sharply and is deeper than the FDA potential around 80 MeV at bombarding energy of 78.8 MeV. We note that from the contact point of Rcut ~ 3.8 fm inward, the calculated potentials depend strongly on the choice of ADA and FDA while both of approximations produce the same potentials outside this point. This means that the ADA and FDA affect critically on the potential strength, which characterizes for distinctive interaction systems. It is now to be seen whether the calculated potentials can describe the angular distributions of ^{12}C + ^{12}C system at energies below 10 MeV per nucleon.

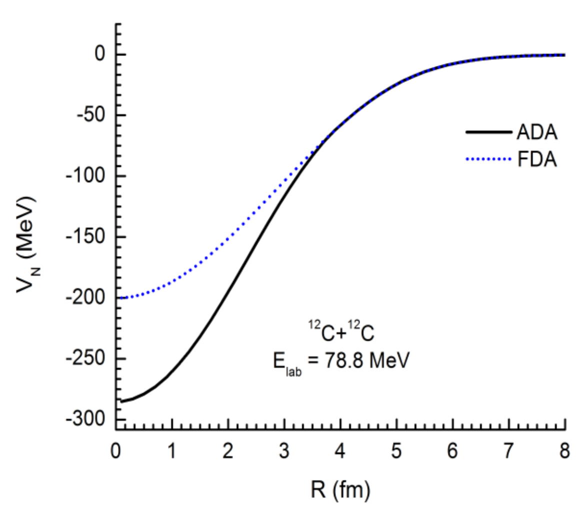

Fig. 2. The real parts of double folding potentials for ^{12}C+ ^{12}C system at the bombarding energy of 78.8 MeV with two frozen and adiabatic density approximations.

To investigate the medium effect during collision process of ^{12}C + ^{12}C system at low energies, the Optical Model calculations are employed to yield the elastic angular distributions that are compared with the experimental data [3]. In this model, the microscopic real potentials corresponding to the frozen and adiabatic density approximations are calculated by using double folding model while the imaginary parts are described by Woods-Saxon shape with parameters adjusted to best fit the measured data, as listed in the table 1. In the Fig.3, angular distribution analyses from optical model calculations, in which the renormalization factor Nr for the real parts of optical potential is equal to 1.0 and the imaginary parts is the same for both two FDA and ADA approximations, are compared with the ^{12}C - ^{12}C elastic scattering data. The results point out that the real parts of potentials calculated by using the ADA describe the data better than that with FDA, especially for the large angles. This means that the potentials yielded from ADA have characters, such as the strength and shape that are relevant to the ^{12}C + ^{12}C realistic interaction potential at low energies. Consequently, the ADA regime is reasonable and realistic to describe the medium effect or the nuclear environment in which two interacting nucleons are embedded at low bombarding energies in comparison with FDA.

Table 1. The parameters of imaginary part of optical potential

Energy (MeV)	W_0 (MeV)	R_I (fm)	a_I (fm)
121.6	4.479	1.403	0.333
83.3	8.675	1.388	0.364
50	16.953	1.214	0.587

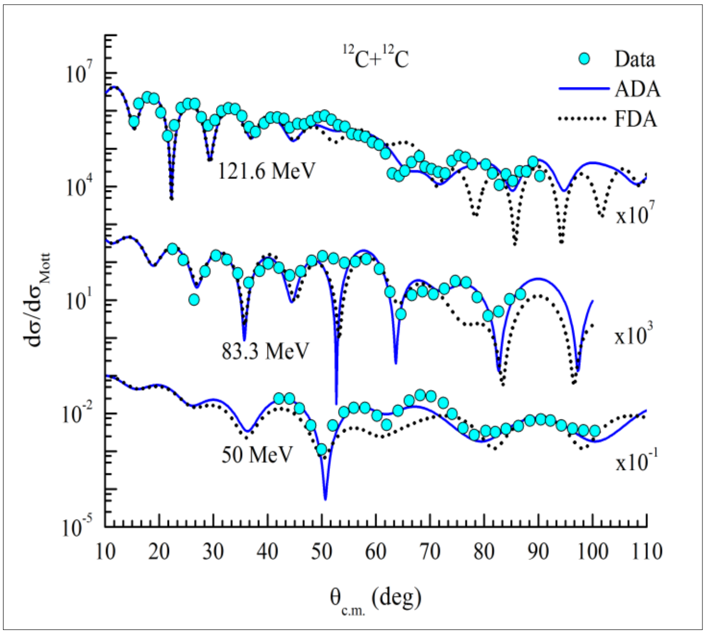

Fig. 3. The elastic angular distributions for ^{12}C + ^{12}C system at low energies. The data are taken from ref. [3].

IV. CONCLUSION

The aim of the work is to investigate the nuclear medium effect during the colliding process of ^{12}C + ^{12}C system at energy region below 10 MeV per nucleon. Both the frozen and adiabatic density approximations are used for describing the nuclear medium. Based on the Optical model potential in which the real parts of the potentials are calculated in the framework of double folding model with the taking into account of medium effect, the elastic scattering angular distributions of ^{12}C + ^{12}C system are analysed and compared with experimental data. The obtained results figure out that the nuclear interacting potentials calculated within adiabatic density approximation yield the angular distributions that is much better fit to data than those yielded by frozen density approximation.

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NGHIÊN CỨU HIỆU ỨNG MẬT ĐỘ HẠT NHÂN DỰA TRÊN PHÂN TÍCH TÁN XẠ ĐÀN HỒI ^{12}C - ^{12}C Ở VÙNG NĂNG LƯỢNG THẤP

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Tóm tắt: Hiệu ứng mật độ hạt nhân được nghiên cứu thông qua phân tích tán xạ đàn hồi ^{12}C - ^{12}C ở vùng năng lượng dưới 10 MeV/nucleon. Trong đó, hai xấp xỉ được sử dụng để nghiên cứu hiệu ứng này là xấp xỉ "frozen" và "adiabatic". Thế hạt nhân trong phân tích này được xây dựng từ mẫu folding kép dựa trên các tương tác hiệu dụng phụ thuộc mật độ CDM3Y3 và hàm sóng trạng thái của các hạt nhân tương tác. Các kết quả thu được từ việc phân tích tán xạ đàn hồi ^{12}C - ^{12}C dựa vào mẫu quang học cho thấy xấp xỉ "adiabatic" mô tả tốt số liệu thực nghiệm hơn so với xấp xỉ "frozen".

Từ khóa: *tán xạ đàn hồi, mẫu quang học, xấp xỉ “adiabatic”.*