THE THEORY OF URANIUM ENRICHMENT BY THE GAS CENTRIFUGE

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Abstract: Relying on the Navier-Stokes equations in cylindrical coordinate and the characteristics of the flow in a gas centrifuge, we derive the set of equations describing the movement and the state of the flow. Basing upon the finite element method to find the appropriate solution of the Onsager's equation (which is also called master potential χ) which was presented by Max D. Gunzburger, a computer program is written by using Maple programming tool to implement the algorithm discussed in the method. In addition, to show the viability of the program, the numerical example for flow driven by a linear temperature gradient along the wall of the centrifuge is presented. Stream function ψ, radial and axial momenta $\rho_0 u$ and $\rho_0 w$ can be deduced from the master potential. These physically variables play an important role in the diffusion-convection equation which is the main part in separation theory. The theoretical analysis of a gas centrifuge provides an understanding of how the flow affects isotope separation and may suggest means of altering the flow profile to improve performance. Such calculations also permit us to optimize the performance of the centrifuge and guide experiments which are used to test the machine.

Keywords: gas dynamics, Onsager's pancake equation, master potential, finite element method.

I. INTRODUCTION

In the context of energy crisis in the world, the nuclear power attracts the interest of many nations. Nowadays, the nuclear power plants provide 13-14% of the world electricity and this percentage grows relentlessly. However, uranium, the fuel for nuclear power plants, is unavailable in the coarse form. Only the very small percentage of U-235 (0.72%) in the uranium ore, the rest is U-238 (99.27%) and U-234 (0.72%) [1]. In a nuclear fission, U-235 plays an important role because of its high cross section in thermal neutron absorption. In order to process uranium ore for nuclear fuel, the percentage of U-235 needs increasing, this is called uranium enrichment. In industry, the gas centrifuge method is most used to enrich U-235 due to its small consuming power compared with other methods. In this method, the U-238 component is separated from U-235 by the difference in centripetal force exerted on them. Therefore, conducting the theoretical analysis of the gas centrifuge is necessary in nuclear fuel research. Such calculations can be used to guide experiments and provide the understanding of how the flow affects isotope separation.

In the theoretical analysis of the gas centrifuge, the Onsager's equation plays an important role. This equation is derived from the continuity, momentum, energy, and state equations for a viscous compressible gas. In centrifuge, it is assumed that rotation rates are sufficiently high so that all the gas is confined to a narrow annulus near the cylinder wall [2]. In addition, it is supposed that the flow represents a small perturbation to the isothermal solid-body rotation [3]. The boundary conditions which are requisite to solve Onsager's equation are also considered.

Searching the solution to the Onsager equation has attracted interest of many scientists. In this text, we present the finite element method proposed by Max D. Gunzburger [2] to find an approximate solution to the equation. The computer program is written by the use of Maple programming tool to implement the algorithm described in the method. In our work, only the homogeneous form of Onsager equation is considered and the solution is presented in the case of linear gradient temperature along the wall.

II. GAS DYNAMICS

1. Perturbations from the equilibrium

Let (r, θ, z) be the cylindrical coordinates with the origin fixed in the bottom of the centrifuge on the axis of rotation. In the absence of mechanical and thermal perturbations of the gas in the rotor, the thermodynamic equilibrium is achieved and the gas rotates as a rigid body.

The pressure ratio or density ratio between an interior radius r and the outer wall of the centrifuge at radius a is:

$$\frac{p(r)}{p_a} = \frac{\rho(r)}{\rho_a} = \exp\left[-A^2\left(1 - \frac{r^2}{a^2}\right)\right] \quad (1)$$

where the dimensionless quantity A is defined by:

$$A^2 = \frac{M\Omega^2 a^2}{2RT_0}$$

where M is molecular weight of the gas, Ω is angular velocity of the centrifuge, R is specific gas constant and T_0 is gas temperature in equilibrium conditions.

The flow field becomes much more complex when circulation currents are generated. Let (u,v,w) be perturbations of the velocity components, and (p′, ρ′, T′) are the corresponding perturbations of the pressure p_{eq}, density ρ_{eq} and temperature T_0 (the subcript "eq" indicates the quantity is considered in equilibrium conditions) [3].

2. Equations of motion of the gas flow in cylindrical coordinates

There are six linearized equations which are derived from conservation equations by linearizing them about equilibrium solution:

$$\frac{1}{r}\frac{\partial(r\rho_{eq}u)}{\partial r} + \frac{\partial \rho_{eq} w}{\partial z} = 0 \quad (2a)$$

$$-\rho' \Omega^2 r - 2\Omega v \rho_{eq} = -\frac{\partial p'}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ur)\right] + \frac{\partial^2 u}{\partial z^2}\right\} \quad (2b)$$

$$2\rho_{eq}\Omega u = \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv)\right] + \frac{\partial^2 v}{\partial z^2}\right\} \quad (2c)$$

$$0 = -\frac{\partial p'}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right\} \quad (2d)$$

$$-\rho_{eq}\Omega^2 ru = k\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right)+\frac{\partial^2 T'}{\partial z^2}\right\} \quad (2e)$$

$$\rho' = p'\frac{M}{RT_0} - \rho_{eq}\frac{T'}{T_0} \quad (2f)$$

3. Onsager's pancake equation:

By the use of equation (1), only regions next to solid boundaries contain gas at significant density. The gas kept close to the rotor wall by the strong centrifugal force flows primarily in axial direction. This circulation forms a boundary layer called Stewartson layer. Onsager showed that the linearized conservation equations can be reduced to a single sixth-order partial differential equation which describes the countercurrent flow in the Stewartson layer. The combining equations (2a), (2b), (2c), (2d), (2e) and (2f) yields:

$$\frac{\partial^2}{\partial x^2}\left\{e^x\frac{\partial^2}{\partial x^2}\left(e^x\frac{\partial^2\chi}{\partial x^2}\right)\right\}+B^2\frac{\partial^2\chi}{\partial y^2}=0 \quad (3)$$

where χ is the master potential and x, y are the nondimensional variables, $x = A^2\left(1-\frac{r^2}{a^2}\right)$ and $y=\frac{z}{a}$. The dimensionless group B is given by: $B=\frac{1}{4}\frac{Re\sqrt{S}}{A^6}$, where Re is Reynolds number. The quantity S is defined: $S = 1 + A^2 Pr\frac{\gamma-1}{2\gamma}$, where Pr is Prandtl number and γ is specific heat ratio.

As we discussed previously, due to the high speed of rotation the gas is confined to the region near the rotor wall. Hence, the Onsager equation is only considered in the domain which is given by the set $D=\{0\le x\le x_T, 0\le y\le y_T\}$, where $y_T=\frac{z_0}{a}$ (z_0 is the height of the rotor) and x_T is chosen to simulate "the top of the atmosphere" at which the gas density becomes extremely small. Equation (3) requires boundary conditions. There are two axial boundary conditions at the bottom end and top caps derived from the Ekman layer analysis and 6 radial boundary conditions [2].

III. SOLUTION TO ONSAGER'S EQUATION

1. Finite element method for Onsager's equation

The finite element method to find the approximate solution to Onsager's equation is presented by Max D. Gunzburger [2]. In his method, the finite-dimensional variational problem is defined by: Seeking a function $\chi_h \in V_h$ such that:

$$B(\chi_h,\phi_h) = C(\phi_h) \quad \text{for all } \phi_h \in V_h \quad (4)$$

where ϕ_h is a test function and:

$$B(\chi,\phi) = \int_0^{y_T}\int_0^{x_T}[(L_3\phi)(L_3\chi)+B^2\phi_y\chi_y]dxdy + 2AB^{3/2}\int_0^{x_T}e^{x/2}\phi_x(x,0)\chi_x(x,0)\,dx + 2AB^{3/2}\int_0^{x_T}e^{x/2}\chi_x(x,y_0)\phi_x(x,y_0)\,dx$$

$$C(\phi) = -\int_0^{y_T}\phi(0,y)f(y)dy + \int_0^{x_T}\{\phi(x,y_T)g_1(x)-\phi(x,0)g_0(x)\}\,dx$$

where L_3 is the differential operator given by: $L_3F = (e^x F_{xx})_x$, $f(y) = \frac{Re}{32A^{10}} \left(\frac{\partial T_a}{\partial y}\right)_{x=0}$, T_a is the gas temperature at the rotor wall, $g_0(x)$ and $g_1(x)$ are quantities in the boundary conditions of the gas at the bottom and top end caps [2].

Gunzburger considered the domain $D = \{0 \le x \le x_T, 0 \le y \le y_T\}$. Partitioning the interval $0 \le x \le x_T$ into M contiguous subintervals $[x_p, x_{p+1}]$, p=0,1,…,M-1, where $0 = x_0 < x_1 < \ldots < x_M = x_T$ and the interval $0 \le y \le y_T$ into N contiguous subintervals $[y_q, y_{q+1}]$, q=0,1,…,N-1, where $0 = y_0 < y_1 < \ldots < y_N = y_T$. The domain D is partitioned into MN rectangles $[x_p, x_{p+1}] * [y_q, y_{q+1}]$.

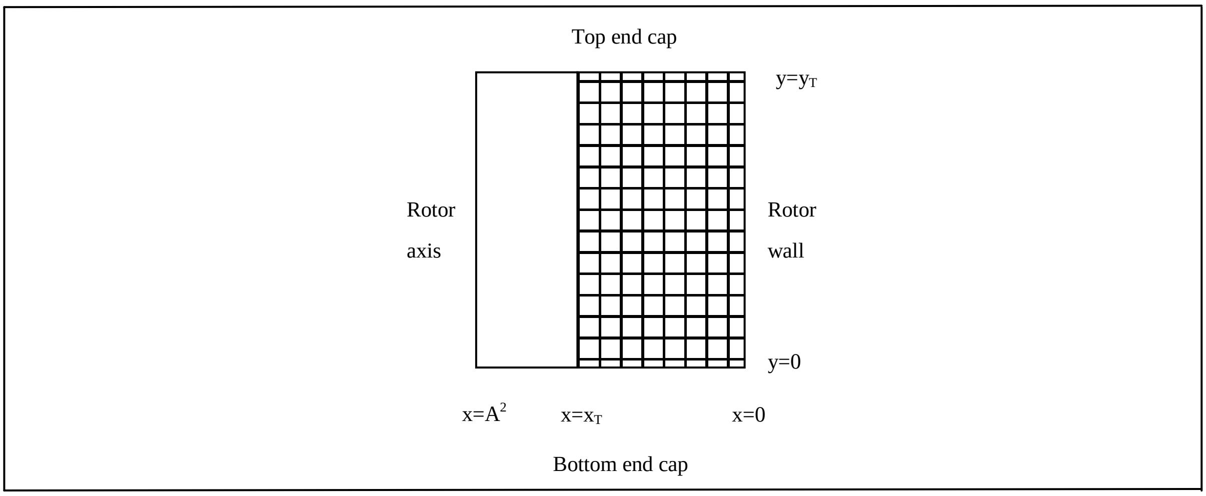

Fig.1. Partition of the intervals $0 \le x \le x_T$ and $0 \le y \le y_T$

Make use of cubic spline functions to construct the set of basis functions of V_h [2]. The solution χ_h can be expressed as follow:

$$\chi_h = \sum_{i=1}^{M-1} \sum_{j=-1}^{N+1} c_k \sigma_i(x) s_j(y) \tag{5}$$

where $\sigma_i(x), s_j(y)$ are the cubic spline functions. The finite-dimensional variational problem (4) is now given by: Seek the solution to a linear system of algebraic equations:

$$Ac = d \tag{6}$$

where c and d are the column vectors in R^K and A is a $K \times K$ matrix.

By the use of equation (6), seeking solution to Onsager's equation is equivalent to determining the matrix c. After obtaining the matrices A and d, the matrix c can be calculated by: $c = A^{-1}d$.

2. Example and program

Table 1 Centrifuge parameters [2], [4]

Parameter	Meaning	Value
z_0	Height of the rotor	3.35 m
A	Radius of the rotor	0.1 m
Ωa	Peripheral speed	700 m/s
T_0	Temperature of the gas in equilibrium conditions	300 K

A	See (1)	5.88
B	See (3)	16.74
Pr	Prandtl number	0.7
Re	Reynolds number	1.94×10^6
γ	Specific heat ratio	1.0935

A computer program has been written to implement the algorithm discussed in the finite element method. The calculations are conducted with the parameters described in Table 1. The wall thermal drive is chosen as the mechanism to generate the countercurrent flow in gas centrifuge. In this case, a linear temperature gradient is applied along the wall of the centrifuge. The difference between the temperature of the top and bottom end caps is 1K and the hotter is the bottom end cap [2].

Some boundary conditions can be simplified as follow:

$$f(y) = -\frac{Re}{32A^{10}}\frac{1}{y_T}$$

$$g_1(x) = g_0(x) \approx 0 \quad (7)$$

The elements A_{lk} of the matrix A and d_l of matrix d can be written in the form [2]:

$$A_{lk} = \int_0^{y_T} s_j(y)s_n(y)dy \int_0^{x_T} L_3\sigma_i(x)L_3\sigma_m(x)dx + \int_0^{y_T} \frac{d}{dy}s_j(y)\frac{d}{dy}s_n(y)\,dy \int_0^{x_T} B^2\sigma_i(x)\sigma_m(x)dx + 2AB^{3/2}\int_0^{x_T}\left[e^{x/2}s_j(y_T)s_n(y_T)\frac{d}{dx}\sigma_i(x)\frac{d}{dx}\sigma_m(x)\right]dx$$

$$+2AB^{3/2}\int_0^{x_T}\left[e^{x/2}s_j(0)s_n(0)\frac{d}{dx}\sigma_i(x)\frac{d}{dx}\sigma_m(x)\right]dx \quad (8)$$

$$d_l = -\int_0^{y_T} f(y)s_n(y)\sigma_m(0)dy + \int_0^{x_T} g_1(x)s_n(y_T)\sigma_m(x)dx - \int_0^{x_T} g_0(x)s_n(0)\sigma_m(x)dx \quad (9)$$

Each integral in A_{lk} need considering as a sum of integrals which are calculated in the domains $[x_p, x_{p+1}]$ or $[y_q, y_{q+1}]$. In each domain, the integral is calculated by Gauss–Legendre quadrature for 2 points.

IV. RESULT AND DISCUSSION

The result of the calculations is presented with M=N=2, x_T=8. In this case, A is a matrix of 5 X 5, d and c is the column vectors of R^5. The computer program's output is:

$$c = \begin{bmatrix} 1.111 \times 10^{-9} \\ 8.799 \times 10^{-10} \\ -7.784 \times 10^{-10} \\ 6.719 \times 10^{-9} \\ -7.504 \times 10^{-9} \end{bmatrix} \quad (10)$$

The axial momentum $\rho_{eq}w$ is determined as follow:

$$\rho_{eq}w = 4A^4\frac{\partial^2\chi}{\partial x^2}$$

$$= 4A^4[\sigma_1(x)]_{xx}[c_1 s_{-1}(y) + c_2 s_0(y) + c_3 s_1(y) + c_4 s_2(y) + c_5 s_3(y)]$$

$$= \begin{cases} -2.4808 \times 10^{-7}x - 2.4808 \times 10^{-7} & \text{for } 0 \leq x < 4 \\ -6.202 \times 10^{-8}x - 2.4808 \times 10^{-7} & \text{for } 4 \leq x < 8 \end{cases} \quad (11)$$

As many materials of the gas centrifuge technique are not published, the gas parameters we used in our program are not completely valid. We try to restrict the error as much as possible by the use of parameters of the gas fairly similar to UF_6 (the parameters of SF_6 employed in the program [4]). In addition, the Gauss–Legendre quadrature for 2 points reduces significantly the program's validity. Moreover, because the procedures to calculate the variables ρw is not completed, its value is only presented with M=N=2. The small number of sub-intervals makes the definitions of this variables vulnerable.

V. CONCLUSION

In our work, the gas dynamics and thermodynamic analyses of the countercurrent flow in a gas centrifuge are reported. Relying on the equations of motion of the gas flow in cylindrical coordinates and the characteristics of the flow in a gas centrifuge, the set of equations describing the movement and the state of the flow is derived. The steps to evolve the Onsager's pancake equation and its boundary conditions are also summarized.

With the help of the computer, the solution to Onsager's equation is derived. We present the result of the program with specific values of the parameters. However, this program has a lot of drawbacks. To obtain a more exact solution to Onsager's equation, we need to obtain the exact parameters of the gas UF_6, involve the feed injection and gas removing and use Gauss-Legendre quadrature for 4 points to calculate the integrals in equation (8) and (9).

The axial momentum $\rho_{eq}w$, which is determined through the solution to Onsager's equation, plays an important role in separation analysis of the gas flow in a centrifuge. This quantity is necessary to solve the axial enrichment equations in enriching and stripping sections. The solution to these equations is used to calculate the separative power (kg U-235/year) , the key to gas centrifuge design.

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LÝ THUYẾT LÀM GIÀU URANIUM BẰNG PHƯƠNG PHÁP LY TÂM

Tóm tắt: Phương trình Navier-Stokes trong hệ tọa độ trụ và các đặc điểm của dòng khí di chuyển trong máy li tâm được sử dụng để rút ra hệ các phương trình mô tả trạng thái và sự chuyển động của dòng khí. Sử dụng phương pháp phần tử hữu hạn của tác giả Max D. Gunzburger ,chúng tôi xây dựng chương trình bằng công cụ Maple để tìm lời giải xấp xỉ cho phương trình Onsager (còn gọi là thế điều khiển χ) trong trường hợp cơ chế điều khiển dòng bằng gradient nhiệt độ tuyến tính. Việc tìm được thế điều khiển χ giúp xác định được hàm dòng ψ, thành phần momen theo phương bán kính $\rho_0 u$ và theo phương trục z $\rho_0 w$, những đại lượng vật lý quan trọng để giải phương trình khuếch tán-đối lưu trong lý thuyết chia tách đồng vị. Việc tính toán lý thuyết giúp chúng tôi mô phỏng hoạt động của máy li tâm từ đó tìm ra những điều kiện tối ưu để việc chia tách đồng vị U^{235} và U^{238} đạt hiệu suất cao nhất.